Universität Karlsruhe (TH)
Mathematisches Institut II
PDoz. Dr. F.Hettlich
Dipl.-Math.techn. S.Ritterbusch
Dr. H.Schon
Prof. Dr. M.Girardi

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Karlsruhe, May 13, 2005

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**Worksheet 5**
**Advanced Mathematics II for Mechanical Engineering**

**Problem 21:** a) Using the Simpson's rule, evaluate an approximate value for

$$\frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2}$$

over $n = 4$ subintervals.

b) Let $n$ be the number of subintervals in the Trapezoidal rule. Determine $n$, such that the remainder term is less than $10^{-2}$ and then, using the Trapezoidal rule and this value of $n$, compute the above integral's approximation.

**Problem 22:** Determine the following integral

$$J(t) = \int_0^1 \arcsin(tx)dx \quad , \quad 0 \le t < 1 \quad ,$$

depending on the parameter $t$ by first calculating the derivative $J'(t)$ in the open interval $0 < t < 1$. Hence determine $J(t)$, $0 \le t < 1$ and using the value of the integral at the point $t = 0$ compute the integration constant. Could $J(t)$ be continuously extended towards $t = 1$?

**Problem 23:** Evaluate $F(s) := \int_0^\infty f(t)e^{-st}dt$ for each of the following functions:

$$\text{a)} f(x) = 3e^{4x} + 2 \qquad \text{b)} f(x) = e^{-x}\cos(2x).$$

For which values of $s$ do those integrals exist?

**Problem 24:** Let the following scalar product be defined in the space of the polynomials of degree 2 over the reals:

$$\langle f, g\rangle := \int_{-1}^{+1} f(x)\cdot g(x)dx.$$

a) Show that the polynomials $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = 3x^2 - 1$ build an orthogonal system and compute $\|P_i\|$, $i = 0, 1, 2$.

b) Represent the polynomial $P(x) = x^2 - x + 1$ as a linear combination of $P_i$, $i = 0, 1, 2$. The coefficients $a_0, a_1, a_2$ should be determined with the help of the scalar product.

**Problem 25:** Expand the following real functions in real trigonometric polynomials:

a) $f(x) = \sin x \cos x$     b) $f(x) = \cos^2 x$
c) $f(x) = \sin^2 x + 1$     d) $f(x) = \cos^3 x$.

**Due date:** Monday, May 30, 2005, 1:00 pm (in the slots outside room 208.1 of the Mathematics Building)