Universität Karlsruhe (TH)
Mathematisches Institut II
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26	27	28	29	30	$\sum$

Karlsruhe, May 23, 2005

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Worksheet 6
Advanced Mathematics II for Mechanical Engineering

Problem 26: Write out the Fourier series for the following 2π periodic function:

$$f(x) = \begin{cases} \cos x, & |x| \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le |x| \le \pi \end{cases}$$

Hint: Sketch $y = f(x)$. Is it either an even function or an odd function. If so, what does this tell you about its Fourier series?

Problem 27: a) Compute the Fourier series of the following function:

$$f(x) = \begin{cases} x(\pi - x), & 0 \le x \le \pi, \\ x(\pi + x), & -\pi \le x \le 0. \end{cases}$$

b) Give the series expansion of $\dfrac{\pi^3}{32}$ by evaluating the Fourier series of the above function at the point $x = \frac{\pi}{2}$.

Problem 28: Determine the complex Fourier series for the 2π periodic function $f(x)$, $x \in \mathbb{R}$:

$$f(x) = \begin{cases} e^{ix} & 0 < x \le \pi \\ 0 & -\pi < x \le 0 \end{cases}.$$

Derive the real Fourier series for the real and imaginary part of f respectively.

Problem 29: Determine the limit of the series

$$\left(\sum_{k=1}^{\infty} \frac{\sin(kL)}{k} \right)$$

($L \in (0, \pi)$) using the Fourier series of the function

$$f(x) = \begin{cases} 0, & -\pi < x < L \\ 1, & L \le x < \pi. \end{cases}$$

Problem 30: The so called "Amplitude modulation" (AM) plays an important role in radio broadcast. In AM the amplitude of a high-frequency "carrier signal" $g(x) = \cos(Nx)$ has its amplitude *modulated* by the amplitude of the message bearing signal $f(x)$, i.e. the resulting oscillation is given by $\hat{f}(x) = f(x)\cos(Nx)$.

a) Show that the Fourier series of the modulated signal $f(x) = a_0 + \sum_{k=1}^{\infty} a_k \cos(kx)$ is

$$\hat{f}(x) = f(x)\cos(Nx) = \frac{a_N}{2} + \frac{1}{2}(a_{2N} + 2a_0)\cos(Nx) + \sum_{\substack{n=1 \\ n \neq N}}^{\infty} \frac{1}{2}(a_{n+N} + a_{|n-N|})\cos(nx).$$

b) Determine the Fourier series for the 2π periodic modulated oscillation $\hat{f} = f \cdot g$ where $f(x) = |x|$ and $g(x) = \cos(40x)$, $-\pi \le x \le \pi$.

Due date: Wednesday, June 1, 2005, 1:00 pm (in the slots outside room 208.1 of the Mathematics Building)