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Karlsruhe, June 06, 2005

Student No.: .....

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**Worksheet 8**
**Advanced Mathematics II for Mechanical Engineering**

**Problem 36:** Determine a fundamental system for the following differential equation

$$u'''(x) - 5u''(x) + 8u'(x) - 4u(x) = 0, \quad x \in \mathbb{R}.$$

Show that the solutions that you found do really build a fundamental system.

**Problem 37:** For the differential equation

$$Ly(x) = x^2y''(x) - 2xy'(x) + 2y(x) = x^3 \ln x, \quad x > 0,$$

determine

a) the general solution of $Ly(x) = 0$ by using reduction of order. Hint: One solution is $y_1(x) = x$;

b) the general solution of $Ly(x) = 0$ using the ansatz $y = x^\alpha$;

c) a particular solution of $Ly(x) = x^3 \ln x$ using the method of variation of constans;

d) the solution of the IVP $Ly(x) = x^3 \ln x$, $y(1) = y'(1) = 1$.

**Problem 38:** Solve the following differential equation $y''' - 6y'' + 9y' = s(x)$ for the perturbation function $s(x)$ :

a) $s(x) = 9$, b) $s(x) = x$, c) $s(x) = e^x$, d) $s(x) = xe^{3x}$, e) $s(x) = 9 + 2x + 3xe^{3x}$.

In each case, the type of the ansatz should match the type of the perturbation function.

**Problem 39:** Given the following differential equation,

$$Ly(x) := y'''(x) + 3y''(x) + 3y'(x) + y(x) = x + 6e^{-x},$$

a) determine the general soluiton of $Ly(x) = 0$.

b) determine a particular solution of $Ly(x) = x + 6e^{-x}$ using an ansatz of the same type as the right hand side and give the general solution of the nonhomogeneous differential equation.

**Problem 40:** Solve the initial value problem (IVP)

$$y''(x) - 4y'(x) + 7y(x) = e^x \sin(x), \quad y(0) = 6/13, \quad y'(0) = 1.$$

**Due Date:** Wednesday, June 15, 2005, 1:00 pm (in the slots outside room 208.1 of the Mathematics Building)