Universität Karlsruhe (TH)
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Karlsruhe, 21.4.2008

Student Nr.: ....

Worksheet No.2
Advanced Mathematics II

Exercise 6: Determine the general solutions of the following differential equations:

$$\text{(a) } y'(x) = \tfrac{1}{1+x^2}, \qquad \text{(b) } y'(x) = x^2 y(x), \qquad \text{(c) } y'(x) = \tfrac{x}{y^2(x)\sqrt{1+x^2}}, \qquad \text{(d) } y'(x) = 1 + \tfrac{y^2(x)}{x^2+xy(x)}.$$

Hint for (d): Use the substitution $z(x) = \frac{y(x)}{x}$. It is enough to give the solution in implicit form.

Exercise 7: Find the general solutions of the differential equations:

(a) $y'(x) - e^{-x} + y(x) - x\,y'(x) = x\,y(x)$,

(b) $y'(x) + y(x) = x\,y^3(x)$.

Exercise 8: Determine the solutions of the following initial value problems:

(a) $y'(x) = \dfrac{1}{1-x} y(x) + x - 1, \quad x > 1, \quad y(2) = 0,$

(b) $y^3(x) - x^2 + xy^2(x)y'(x) = 0, \quad x > 0, \quad y(1) = 1,$

(c) $y'(x) = \sqrt{1 - y^2(x)}, \quad y(0) = \dfrac{1}{2}.$

Exercise 9: Show that the following initial value problem has infinitely many solutions:

$$y'(x) = \sqrt{y(x)}, \quad y(0) = 0.$$

Hint: Combine the trivial solution and the general solution of the equation solved as an equation with separated variables.

Exercise 10: Find all continuous differentiable functions $y : \mathbb{R}\backslash\{0\} \to \mathbb{R}$ satisfying the following condition: The tangent of y at the point $(x_0, y(x_0))$ intersects the y-axis in the point $(0, -\frac{y(x_0)^3}{x_0^2})$.

Due date: Please hand in your homework on Wednesday, April 30, 09:30.

Tutorial 1
Advanced Mathematics II

Aufgabe T1: Determine the general solution of the following differential equations:

(a) $y'(x) = \dfrac{2^x}{y(x)}$ (b) $y'(x) - 1 = y(x)x - y(x) - x$ (c) $y'(x) = \dfrac{y^2(x) + 1}{x^2 + 6x + 10}$

Aufgabe T2: Determine the type and the solution of the following initial value problem:

(a) $y'(x) = 2\frac{y(x)}{x}$ with $y(1) = 2$,

(b) $y'(x) = 2\frac{y(x)}{x} + x$ with $y(1) = 2$,

(c) $y'(x) = \frac{y(x)}{x} + \frac{x}{2y(x)}$ with $y(1) = \sqrt{2}$.

Aufgabe T3: Solve the initial value problem

$$y'(x) + y(x) - y^3(x) = 0, \quad y(0) = \frac{1}{2}.$$

Aufgabe T4: Show that each solution of an autonomous differential equation $u'(x) = h(u(x))$ is translation invariant, i.e. if $u(x)$ is a solution then $v(x) := u(x + a)$ with $a \in \mathbb{R}$ is also a solution. Solve the differential equation in the case when $h(u) = u(u - 1)$.

Hint: $\frac{1}{z(z-1)} = \frac{1}{z-1} - \frac{1}{z}$.

Tutorial date: Wednesday, April 23.