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Karlsruhe, 21.4.2008

Student Nr.: ..................

# Worksheet No.3
# Advanced Mathematics II

**Exercise 11:** Compute the following definite integral by means of partial fraction decomposition,

$$\int_0^{\frac{1}{2}} \frac{(x-2)\,dx}{x^3+2x^2-x-2} \quad .$$

**Exercise 12:** Evaluate the following indefinite integral using the substitution $u(x) = \tan(\frac{x}{2})$,

$$\int \frac{2}{\tan(\frac{x}{2}) + \cos(x) - \sin(x)} dx.$$

**Exercise 13:** We consider the following sequence of functions

$$f_k(x) = e^{-\frac{x^2}{2}} \sum_{m=0}^{k} \frac{x^{2m+1}}{2^{2m}(m!)^2}, \quad x \geq 0, \quad k = 0, 1, 2, \ldots$$

(a) Show that this sequence of functions is monotonically increasing, that means $f_k(x) \leq f_{k+1}(x)$, $x > 0$.

(b) Use the equation $\int\limits_0^\infty x^{2m+1} e^{-\frac{x^2}{2}}\, dx = 2^m m!$ to show the existence of a positive number $M$ independent of $k$ which satisfies the inequality $\int_0^\infty f_k(x)dx < M$.

(c) Prove the equation $\int\limits_0^\infty \lim\limits_{k\to\infty} f_k(x)dx = \sqrt{e}$ by using the Beppo-Levi theorem.

**Exercise 14:** Prove that

(a) the following improper integral exists if $\alpha > -2$: $\int\limits_0^1 x^\alpha \sin \frac{1}{x}\, dx$ but

(b) that it does not exist for $\alpha = -2$, that means $\lim_{A\to 0} \int\limits_A^1 x^{-2} \sin \frac{1}{x}\, dx = \infty$.

Hint: $x^\alpha = x^{\alpha+2} \dfrac{1}{x^2}$ and $(\cos(x^{-1}))' = \sin(x^{-1})x^{-2}$.

**Exercise 15:** A pool of water is supplied through four feed pipes. At time $t = 0$ the pool contains 100 liters of water. Afterwards, $\frac{\ln 2}{2^t}$ liters of water per second [l/sec] flow through pipe number one into (or out of) the pool, via the second pipe there stream $9te^{-3t} + \frac{2}{1+t^2}$ l/sec. For $0 \leq t \leq 4$ via the third pipe pour $\frac{-1}{4\sqrt{t}}$ l/sec. Via the last pipe we have a stream of $\frac{1}{t^2}$ l/sec for $t \geq c$. Determine the amount of water in the pool for $t \to \infty$.

**Due date:** Please hand in your homework on Wednesday, May 7, 09:30.

## Tutorial 2
## Advanced Mathematics II

**Aufgabe T5:** Evaluate the integral by means of partial fraction decomposition:

$$\int \frac{x^3+6x^2+3x+18}{x^3+x^2+4x+4}\,dx.$$

**Aufgabe T6:** Determine the anti-derivative by means of an appropriate substitution:

$$\int \frac{5e^{3x}+e^{2x}+3e^x+1}{e^{3x}+e^x}dx.$$

**Aufgabe T7:** Investigate the existence of the following improper integrals and if necessary determine its value.

(a) $\displaystyle\int_0^{\pi/2} \tan x\,dx$ (b) $\displaystyle\int_2^{\infty} \frac{x+2}{x^3-x^2-x+1}dx$ (c) $\displaystyle\int_{-\infty}^{0} \frac{e^x}{1+e^x}dx.$

**Aufgabe T8:** Prove that

(a) the following improper integral exists

$$\int_2^{\infty} \frac{dx}{x(\ln x)^\alpha}$$

if $\alpha > 1$ and

(b) in does not exist for $\alpha = 1$, that means $\lim_{A\to\infty} \int_2^A (x\ln(x))^{-1}\,dx = \infty$.

**Tutorial date:** Wednesday, April 30.