Universität Karlsruhe (TH)
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16	17	18	19	20	Σ

Karlsruhe, 5.5.2008

Student Nr.: ....

Worksheet No.4
Advanced Mathematics II

Exercise 16: Compute an anti derivative of

$$f(t) = \frac{2(\tan(t))^4 + 3(\tan(t))^3 + (\tan(t))^2 - 2}{\cos(t)(\tan(t))^3 + \cos(t)(\tan(t))^2 - \sin(t) - \cos(t)}.$$

Hint: Start with a suitable substitution – the $\tan(t/2)$-substitution is rather complicated here.

Exercise 17: Let the function $g : (0, \infty) \to \mathbb{R}$ be continuously differentiable, strictly monotonously increasing and onto. Determine the general solution of the differential equation

$$u'(x) = -\frac{x^2}{x^2 - 1} \frac{g(u(x))}{g'(u(x))}, \qquad x > 0.$$

Formulate this differential equation explicitely for the case $g(x) = \ln(x)$ and determine the solution which satisfies the initial value condition $u(0) = e$. Finally, considering again $g(x) = \ln(x)$, compute all values $a \in \mathbb{R}$ such that the initial value problem with initial condition $u(1) = a$ has a solution.

Exercise 18: Compute the general solution of the differential equation

$$y^{(4)}(x) - 7y'''(x) + 18y''(x) - 20y'(x) + 8y(x) = 0.$$

Exercise 19: Determine the real general solution of the differential equation

$$u'''(x) + 3u''(x) + 9u'(x) - 13u(x) = 0, \quad x \in \mathbb{R}.$$

Show by means of the general solution that every initial value problem $u(0) = a$, $u'(0) = b$, $u''(0) = c$ has a unique solution.

Exercise 20: Consider the inhomogeneous linear differential equations

$$\text{(a) } y''(x) - 2y'(x) - 3y(x) = x^2 + \frac{4}{3}x + \frac{4}{3} \quad \text{and} \quad \text{(b) } y'''(x) - By'(x) + y(x) = \sin(x), \quad B \in \mathbb{R}.$$

Find a polynomial of degree two which solves the differential equation in (a), and a trigonometric polynomial $p(x) = a_0 + a_1 \cos(x) + b_1 \sin(x)$ which solves the equation in (b). Finally, compute the constant B appearing in (b) such that $f(x) = \sin(x)$ solves the differential equation in (b).

Due date: Please hand in your homework on Wednesday, May 14, 09:30.

Tutorial 3
Advanced Mathematics II

Aufgabe T9: (a) Determine the general solution of the differential equation:

$$y'''(x) + 3y''(x) + 4y'(x) + 2y(x) = 0$$

using the ansatz $y(x) = e^{\lambda x}$, $\lambda \in \mathbb{C}$.

(b) Afterwards, determine numbers $a, b, c \in \mathbb{R}$ such that the polynomial $p(x) = ax^2 + bx + c$ solves the corresponding inhomogeneous differential equation for right hand side $x^2 + 11$.

Aufgabe T10: Determine the general real-valued solution of the homogeneous linear ordinary differential equation of third order

$$y'''(x) + 2y''(x) + 2y'(x) + y(x) = 0$$

using the exponential ansatz $y(x) = e^{\lambda x}$, $\lambda \in \mathbb{C}$.

Aufgabe T11: Determine for each of the following differential equations a real fundamental system:

(a) $y^{(6)}(x) + y^{(4)}(x) - 5y^{(2)}(x) + 3y(x) = 0$

(b) $y^{(8)}(x) - y^{(7)}(x) + 2y^{(6)}(x) - 2y^{(5)}(x) + y^{(4)}(x) - y^{(3)}(x) = 0.$

Aufgabe T12: Determine a real-valued fundamental system for the homogeneous fourth-order ordinary differential equation

$$y^{(4)}(x) - 3y'''(x) + 4y'(x) = 0 \, .$$

Tutorial date: Wednesday, Mai 7.