Universität Karlsruhe (TH)
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21	22	23	24	25	Σ

Karlsruhe, 13.5.2008

Student Nr.: ....

Worksheet No.5
Advanced Mathematics II

Exercise 21: Determine for each of the following differential equations a real fundamental system:

(a) $9u'(x) + 11xu^{(2)}(x) + 4x^2u^{(3)}(x) + x^3u^{(4)}(x) = 0$

(b) $-8u(x) + 8xu'(x) + 28x^2u^{(2)}(x) + 11x^3u^{(3)}(x) + x^4u^{(4)}(x) = 0$

Exercise 22: Determine the general solution of the following differential equation

$$2x^2z''(x) + 4x^2\left[z'(x)\right]^2 + 6xz'(x) + 5 = 0, \qquad x > 0.$$

Hint: Use the substitution $y(x) = e^{2z(x)}$.

Exercise 23: Consider the differential equation

$$2x^3u'''(x) + Bx^2u''(x) + xu'(x) - 10u(x) = 0, \quad x > 0.$$

(a) Determine B so that $u_1(x) = x^{\frac{5}{2}}$ is a solution of the differential equation.

(b) With the constant B from part (a) determine the general solution of the differential equation.

Exercise 24: Consider the homogeneous linear differential equation with non-constant coefficients

$$(1 + x^2)u''(x) - 2xu'(x) + 2u(x) = 0, \quad x \in (0, \infty).$$

(a) Verify that $u(x) = x$ is a solution of the differential equation.

(b) Determine by reduction of order a solution $u_2(x)$ that is linearly independant of $u_1(x)$.

(c) The real-valued general solution is $u(x) = C_1u_1(x) + C_2u_2(x)$ where $C_1, C_2 \in \mathbb{R}$. Show that each initial value problem $u(1) = a$, $u'(1) = b$ with $a, b \in \mathbb{R}$ has a unique solution.

Exercise 25: Determine the general solution of the nonlinear differential equation

$$1 + [y'(x)]^2 = 2y(x)y''(x).$$

Hint: Consider x as function of y. Define the function p with $p(y) = y'(x(y))$ and compute $p'(y)$. In this way one can reduce the order of the differential equation.

Due date: Please hand in your homework on Wednesday, May 28, 09:30.

Tutorial 4
Advanced Mathematics II

Aufgabe T13: Determine the real-valued general solution of the homogeneous third-order ordinary differential equation

$$u'''(x) - \frac{2}{x}u''(x) + \frac{5}{x^2}u'(x) - \frac{5}{x^3}u(x) = 0, \quad x > 0.$$

Aufgabe T14: Find the real-valued general solution for each of the following homogeneous ordinary differential equations:

(a) $x^2u''(x) - 5xu'(x) + 13u(x) = 0$, $x > 0$,

(b) $u''(x) - 5u'(x) + 13u(x) = 0$, $x > 0$,

(c) $u'''(x) - \frac{3}{x}u''(x) + \frac{7}{x^2}u'(x) - \frac{8}{x^3}u(x) = 0$, $x > 0$.

Aufgabe T15: Consider the differential equation

$$(1-x)y''(x) + xy'(x) - y(x) = 0.$$

(a) Check that $y(x) = x$ is a solution.

(b) Determine the solution of the following initial value problem

$$y(0) = 1, \quad y'(0) = 3.$$

Hint: Via reduction of order determine first a solution $y_2(x)$ that is linearly independant of $y_1(x)$.

Aufgabe T16: Determine the solution of the following initial value problem

$$y''(x) = 2y(x)y'(x), \quad y(0) = 1, \quad y'(0) = 2.$$

Hint: Consider x as function of y. Define the function p with $p(y) = y'(x(y))$ and compute $p'(y)$. In this way one can reduce the order of the differential equation.

Tutorial date: Wednesday, May 14.