Universität Karlsruhe (TH)
Institut für Algebra und Geometrie
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26	27	28	29	30	$\sum$

Karlsruhe, 19.5.2008

Student Nr.: ....

Worksheet No.6
Advanced Mathematics II

Exercise 26: Consider the following differential equation

$$u''(x) + Bu'(x) + 4u(x) = 8\sin(x)\cos(x), \quad B \in \mathbb{R}.$$

(a) Determine B so that $u_p(x) = 4\cos^2(x) - 2$ is a solution.

(b) With the constant B from part (a) determine the real-valued general solution of the differential equation.

Exercise 27: Solve the initial value problem

$$y'''(x) + 3y''(x) + 4y'(x) - 8y(x) = (7 - 13x)e^x$$

with $y(0) = y''(0) = 2$ and $y'(0) = 0$.

Exercise 28: For the nonhomogeneous linear differential equation of 2nd order

$$y''(x) - 3\,y'(x) - 2y(x) = -18x\sin(x), \quad x \in \mathbb{R},$$

find a particular solution by the method of undetermined coefficients.

Exercise 29: Consider the nonhomogeneous linear differential equation

$$(1 + x^2)u''(x) - 2xu'(x) + 2u(x) = 3x + x^3, \quad x \in (0, \infty).$$

Determine the general solution of the differential equation by the method of variation of parameters.

Hint: You can use that $\{x, x^2 - 1\}$ is afundamental system of the homogeneous differential equation.

Exercise 30: Consider the following differential equation

$$4u(x) - 2xu'(x) + x^3u^{(3)}(x) = 9, \quad x > 0.$$

(a) Determine a real fundamental system.

(b) Determine a particular solution by the method of variation of parameters.

(c) Solve the initial value problem with $u(1) = \frac{13}{4}$, $u'(1) = \frac{27}{4}$ and $u^{(2)}(1) = \frac{50}{4}$.

Due date: Please hand in your homework on Wednesday, May 28, 09:30.

Tutorial 5
Advanced Mathematics II

Aufgabe T17: Determine the general solution of the differential equation

$$y''(x) - 4y'(x) + 13y(x) - (10x - 2)e^x = 0\,.$$

Aufgabe T18: Determine the roots of the characteristic polynomials and an ansatz for a particular solution by the method of undetermined coefficients of the following differential equations:

(a) $y''(x) + y(x) = x \sin x$,

(b) $y'''(x) - 4y''(x) - 2y'(x) + 20y(x) = x^2 e^x$,

(c) $y'''(x) + 6y''(x) + 12y'(x) + 8y(x) = xe^{-2x}$,

(d) $y'''(x) + y''(x) - 6y'(x) = xe^{2x} + 2e^{-3x}$,

(e) $y^{(4)}(x) + 4y'''(x) + 6y''(x) + 4y'(x) + 5y(x) = -8\cos x - 8 \sin x$,

(f) $y^{(5)}(x) + y^{(4)}(x) - 4y'''(x) - 16y''(x) - 20y'(x) - 12y(x) = e^{-3x}$.

Hint: A solution of the homogeneous differential equation in (e) is $y(x) = x\cos(x)$ and one solution of the homogeneous differential equation in (f) is $y(x) = x\sin(x)e^{-x}$.

Aufgabe T19: Consider the linear non-homogeneous differential equation

$$(1 - x)y''(x) + xy'(x) - y(x) = (1 - x)^2.$$

Determine the general solution by the method of variation of parameters.

Hint: You can use that $\{x, e^x\}$ is a fundamental system.

Aufgabe T20: Consider the inhomogeneous linear second-order ordinary differential equation

$$-15u(x) + 3xu'(x) + x^2u''(x) = 8x^{-3}, \qquad x > 0.$$

(a) Find a real-valued fundamental system of the associated homogeneous differential equation.

(b) Find a particular solution by the method of variation of parameters. Determine the general solution of the inhomogeneous problem.

Tutorial date: Wednesday, May 21.