Universität Karlsruhe (TH)
Institut für Algebra und Geometrie
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31	32	33	34	35	$\sum$

Karlsruhe, 19.5.2008

Student Nr.: ....

Worksheet No.7
Advanced Mathematics II

Exercise 31: For the inhomogeneous linear second-order ordinary differential equation

$$x^2y''(x) - \frac{3}{2}xy'(x) + y(x) = x^3\ ,$$

determine

(a) the general solution of the homogeneous differential equation by means of reduction of the order. Use the fact that $y_1(x) = x^2$ solves the homogeneous problem.

(b) a particular solution and the general solution of the inhomogeneous differential equation by means of variation of the constants.

(c) a solution of the initial value problem with $y(1) = \frac{17}{5}$ and $y'(1) = \frac{21}{5}$.

Exercise 32: Determine the general solution in real form by the method of undetemined coefficients:

$$y''(x) - 4y'(x) + 13y(x) = 2e^{2x}\sin(3x).$$

Exercise 33: Solve the initial value problem

$$xu''(x) + 4u'(x) + 3u(x) = 3, \quad u(0) = 2,$$

with the power series method. For which $x \in \mathbb{R}$ does the series converge absolutely? You don't have to give the solution in an explicit form.

Exercise 34: Solve the initial value problem

$$(2x - x^2)y''(x) + (1 - x)y'(x) = 0, \quad y(1) = 1, \quad y'(1) = 0$$

with the power series method.

Exercise 35: Determine the general solution of the differential equation

$$x^2y''(x) + x^3y'(x) - 6y(x) = 0\,.$$

Use the generalized power series representation $y(x) = \sum_{k=0}^{\infty} a_k x^{k+\lambda}$.

Due date: Please hand in your homework on Wednesday, June 4, 09:30.

Tutorial 6
Advanced Mathematics II

Aufgabe T21: Let the following differential equation be given

$$x^2 y''(x) - 2xy'(x) + 2y(x) = x^3 \ln x, \quad x > 0.$$

The corresponding homogeneous differential equation has a solution of the form $y(x) = Ax + B$.

(a) Determine the general solution of the homogeneous problem by reduction of order.

(b) Determine a particular solution of the nonhomogeneous problem by variation of constants.

(c) Solve the initial value problem of the nonhomogeneous differential equation with $y(1) = y'(1) = 1$.

Aufgabe T22: Solve the initial value problem by the method of undetermined coefficients:

$$u'''(t) + 4u'(t) = -2e^{-t}, \quad u(0) = 0, \quad u'(0) = 0, \quad u''(0) = 2.$$

Aufgabe T23: Determine the general solution of the homogeneous linear second-order ordinary differential equation

$$u''(x) + x^2 u'(x) + 2xu(x) = 0$$

using a power series ansatz near $x_0 = 0$ and specify the domain of convergence.
Remark: One of the two fundamental solutions can be expressed in terms of elementary functions.

Aufgabe T24: Solve the differential equation

$$(2 + x)y''(x) + y'(x) = 1$$

by means of an ansatz in power series form with center of expansion $x_0 = 0$ and determine the domain of convergence.

Tutorial date: Wednesday, May 28.