Universität Karlsruhe (TH)
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Karlsruhe, 02.06.2008

Student Nr.: ....

Worksheet No.8
Advanced Mathematics II

Exercise 36: Determine the general solution of the following differential equation

$$u'''(x) - 2u''(x) + u'(x) = 1 + x^2 + e^x, \quad x > 0.$$

Exercise 37: Determine the general solution of the following differential equation

$$x^2 y''(x) + xy'(x) - 4y(x) = 1 + x^2, \quad x > 0.$$

Exercise 38: Consider Euler's method for numerically solving the IVP $y'(x) = 1 - x + y(x)$, $y(x_0) = y^{(0)}$. The idea of this exercise is to show that for decreasing step size $h > 0$ the numerical solution converges pointwise to the exact solution.

(a) First find the exact solution of the given IVP.

(b) Define y_k to be the approximate solution obtained from Euler's method at $x_k = x_0 + kh$. Show that $y_k = (1+h)^k (y^{(0)} - x_0) + x_k$.

(c) Choose $x > x_0$ to be arbitrary but fixed, and set the step size to $h = (x - x_0)/k$ for $k \in \mathbb{N}$. Then the approximate solution obtained from Euler's method at $x_k = x$ is given by y_k. Show that for $k \to \infty$ one has $y_k \to y(x)$. You may use that $\lim_{k\to\infty}(1 + a/k)^k = \exp(a)$.

Exercise 39: Draw the graph of the function $z = f(x, y) = x^2 + y^2$,

(a) by determining and drawing the curves that are intersection of the graph with the planes $x = x_0, x_1, \ldots$ and $y = y_0, y_1, \ldots$ respectively,

(b) by determining and drawing the level curves $z = c_0, c_1, \ldots$,

(c) by showing that the graph is a surface of revolution, i. e. a surface created by rotating a curve around an axis of rotation.

Exercise 40: Consider the functions $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = e^x y$, and $g : \mathbb{R}_{>0} \to \mathbb{R}$, $g(t) = \ln(t)$.

(a) Determine the level curves of f.

(b) Draw the graph of g.

(c) Determine the domain and range of $h(x, y) := g(f(x, y))$.

(d) Explain why the composition $f(g(t))$ does not make sense.

Due date: Please hand in your homework on Wednesday, June 11, 09:30.

Tutorial 7
Advanced Mathematics II

Aufgabe T25: Determine the general solution of the following differential equations

(a) $y''(x) - y(x) = x,$ (b) $y''(x) - y(x) = \frac{1}{x}, \quad x \neq 0.$

Hint: The function e^x/x has no simple closed-form anti derivative.

Aufgabe T26: Assign to each differential equation (a)-(h) possible solution methods. Explain your decision.

(a) $y'(x) = c(x)$

(b) $-6x^2y''(x) + 15xy'(x) - 15y(x) = 0$

(c) $y(x) = a_1y'(x) + a_2y''(x)$

(d) $y(x) = y'(x)$

(e) $a_3x^3y'''(x) + a_1xy'(x) = 0$

(f) $y(x) = a_1y'(x) + a_2y''(x) + b(x)$

(g) $x^2y''(x) - 2y(x) = x$

(h) $y'(x) = xy^2(x).$

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Integration								
Separation of variables								
Power series method								
Method of variation of parameters								
Method of undetermined coefficients								
Exponential ansatz								
Power ansatz								

Tabelle 1: Gewöhnliche Differentialgleichungen und ihre Lösungsansätze

Aufgabe T27: Consider the functions $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$, und $g : \mathbb{R}_{\geq 0} \to \mathbb{R}$, $g(t) = \sqrt{t}$. Determine $f(g(x))$, $g(f(x))$ and their domains and ranges.

Aufgabe T28: Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = xy$. Determine

(a) the level curves of f,

(b) the intersection curves with the planes $x = x_0$ and $y = y_0$ with $x_0, y_0 \in \mathbb{R}$

(c) the intersection curves with the planes $x + y = 0$ and $x - y = 0$. How does the graph of $f(g(x))$ with $g : \mathbb{R} \to \mathbb{R}^2$, $g(x) = (x, x)$ look like?

Tutorial date: Wednesday, June 4.