Universität Karlsruhe (TH)
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36	37	38	39	40	$\sum$

Karlsruhe, 9.06.2008

Student Nr.: ....

Worksheet No.9
Advanced Mathematics II

Exercise 36: Given the 3 vectors

$$b_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \qquad b_2 = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}, \qquad b_3 = \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}$$

in $\mathbb{R}^3$, determine the matrix A of the linear map Φ with $\Phi(e_1) = b_1,\ \Phi(e_2) = b_2,\ \Phi(e_3) = b_3$, where e_j denotes the jth coordinate unit vector. Also given the 3 vectors

$$c_1 = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}, \qquad c_2 = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}, \qquad c_3 = \begin{pmatrix} -5 \\ 0 \\ 7 \end{pmatrix},$$

show that the matrix

$$B = \begin{pmatrix} 0 & 1 & 0 \\ 7 & 4 & 5 \\ 4 & 2 & 3 \end{pmatrix}$$

defines a linear map $\Psi : x \mapsto Bx$, with $Bc_j = e_j$, $j = 1, 2, 3$. Also show that the linear map $\Lambda : x \mapsto (AB)x$ satisfies $\Lambda c_j = b_j$, $j = 1, 2, 3$.

Exercise 37: For 3 times kitchen duty (K) and once swabbing the deck (S), poor seaman Hein B. obtains one Euro (E) and a pudding (P) from his captain. Moreover, Hein gets a pudding for four hand made fishing rods (R) and once swabbing the deck. For two fishing rods and four fish (F) he can afford a lemonade (L) in the harbour bar. Finally, for α fish and one kitchen duty, his captain tells one of his thrilling cock-and-bull stories (G). Here, $\alpha \in \mathbb{R}$ is a parameter depending on the captain's mood. Find the matrix A_α representing the linear mapping $(E, P, L, G) \mapsto (K, S, R, F)$. For which α is this mapping invertible? If it is invertible, compute the inverse and describe its meaning for kitchen duty, swabbing, fishing rods and fish.

Exercise 38: Consider the subspaces $E = \{x \in \mathbb{R}^3 : -x_1 + x_3 = 0\}$ and $F = \{x \in \mathbb{R}^3 : x_1 = 0\}$.

(a) Compute the transformation matrices A and B corresponding to reflections with respect to E and F, respectively.

(b) What rotation about what axis does the matrix AB correspond to?

Exercise 39: Compute the following determinants:

$$\text{(a) } D = \begin{vmatrix} 3 & \frac{3}{7} & 2 & \pi \\ 0 & 1 & a & 4 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{vmatrix}, \qquad \text{(b) } D = \begin{vmatrix} 2 & 4 & 2 & -1 \\ 2 & 3 & 0 & 5 \\ 2 & 1 & 2 & 3 \\ 1 & 2 & 0 & 2 \end{vmatrix}, \qquad \text{(c) } D = \begin{vmatrix} 3 & 2 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ -1 & 2 & 0 & 3 \\ 6 & 2 & 3 & 1 \end{vmatrix}.$$

Exercise 40: Consider the matrix

$$A_t = \begin{pmatrix} 1 & 0 & -2 & 3 \\ -1 & 2 & -t & -1 \\ -2 & t-1 & 4 & -1 \\ 1 & 0 & -2 & t \end{pmatrix} \in \mathbb{R}^{4\times 4}.$$

For which $t \in \mathbb{R}$ does the linear system of equations $A_t x = 0$ only possess the trivial solution? Hint: Use the determinant!

Due date: Please hand in your homework on Wednesday, June 18, 09:30.

Tutorial 8
Advanced Mathematics II

Aufgabe T25: The 3 vectors $b^{(1)} = (1,3,1)^\top$, $b^{(2)} = (-2,2,1)^\top$, $b^{(3)} = (1,-4,1)^\top$ in $\mathbb{R}^3$ are given. Determine the matrix A of the linear map Φ with $\Phi(e^{(1)}) = b^{(1)}$, $\Phi(e^{(2)}) = b^{(2)}$, $\Phi(e^{(3)}) = b^{(3)}$, where $e^{(j)}$ denotes the jth coordinate unit vector.

Also consider the 3 vectors $c^{(1)} = (1,-3,4)^\top$, $c^{(2)} = (-1,4,-5)^\top$, $c^{(3)} = (0,-2,3)^\top$. Show that the matrix

$$B = \begin{pmatrix} 2 & 3 & 2 \\ 1 & 3 & 2 \\ -1 & 1 & 1 \end{pmatrix}$$

defines a linear map $\Psi : x \mapsto Bx$ with $Bc^{(j)} = e^{(j)}$, $j = 1,2,3$.

Aufgabe T26: Evaluate the matrix products AB^*, $BC^\top$ and $C^\top BA^\top$ for

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2i & 0 \\ 3 & 0 & 3 \\ 0 & 4i & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \\ 4 & 3 & 2 \\ 5 & 4 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} i & 0 & 0 \\ 2 & 2i & 0 \\ 3i & 3 & 3i \\ 0 & 4i & 4 \\ 0 & 0 & 5i \end{pmatrix}.$$

Aufgabe T27: Compute the determinant of the matrix $C := (AB)^{-1}$ with

$$A := \begin{pmatrix} 1 & 2 & 3 & -2 & 2 \\ 3 & -1 & 4 & 2 & 0 \\ 8 & -3 & -2 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } B := \begin{pmatrix} -1 & 3 & -2 & 2 & -1 \\ 0 & 2 & -3 & -3 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}.$$

Aufgabe T28: Compute the determinant of the matrix

$$A_\alpha = \begin{pmatrix} 3 & \alpha-1 & -\alpha+3 \\ 0 & \alpha+3 & -4 \\ 0 & 2 & \alpha-3 \end{pmatrix}.$$

For which α is the linear system of equations $A_\alpha x = b$ for $b = (9,4,-2)^\top$ solvable?

Tutorial date: Wednesday, June 11.