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Karlsruhe, 16.06.2008

Student Nr.: ....

Worksheet No.10
Advanced Mathematics II

Exercise 46: Determine the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 3 & 0 & 2 \\ 0 & 1 & a \\ 0 & 2 & 2a \end{pmatrix}$ mit $a \in \mathbb{R}$.

Exercise 47: Consider the following differential equation

$$u'''(x) - u''(x) + 4u'(x) - 4u(x) = 0.$$

Transform the differential equation to an equivalent system of first order. Determine the complex-valued general solution of this system (and the real-valued solution respectively) and deduce from it the complex-valued general solution of the initial differential equation (and the real-valued one respectively.)

Exercise 48: Consider the linear system of differential equations

$$x'(t) = Ax(t) \quad \text{für} \quad A = \begin{pmatrix} 8 & -1 \\ 4 & 12 \end{pmatrix}.$$

(a) Which of the following functions are solutions of the linear system of differential equations:

$x_1(t) = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{10t}$, $\quad x_2(t) = \begin{pmatrix} 1 \\ -2 \end{pmatrix} te^{10t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{10t}$, $\quad x_3(t) = \begin{pmatrix} 1 \\ -2 \end{pmatrix} te^{10t}$,

$x_4(t) = \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{10t}$, $\quad x_5(t) = \begin{pmatrix} 1 \\ -2 \end{pmatrix} te^{10t} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{10t}$?

(b) Determine with the help of the Wronskian which of the solutions form a fundamental system.

Exercise 49: Consider the matrix $A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 5 & 6 \\ 0 & -3 & -4 \end{pmatrix}$.

(a) Compute the eigenvalues of A and a basis $\{v^1, v^2, v^3\}$ of $\mathbb{R}^3$ which consists only of eigenvectors.

(b) Let P be the matrix with columns v^1, v^2 und v^3. Show that the matrix $D := P^{-1}AP$ is of the form $\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$ where λ_1, λ_2, λ_3 are the eigenvalues of A.

Exercise 50: (a) Determine the eigenvalues of $A = \begin{pmatrix} 5 & 0 & 4 \\ 0 & -6 & 0 \\ 1 & 0 & 2 \end{pmatrix}$.

(b) Let $p(x) = c_3x^3 + c_2x^2 + c_1x + c_0$ be the characteristic polynomial of A. Show that $p(A) = 0$, i.e. $c_3A^3 + c_2A^2 + c_1A + c_0I_3 = 0$. (This is true in general for every square matrix and its characteristic polynomial!)

(c) From $p(A) = 0$, determine the inverse matrix A^{-1}.

Due date: Please hand in your homework on Wednesday, June 25, 09:30.

Tutorial 9
Advanced Mathematics II

Aufgabe T33: Determine all eigenvalues of the square matrices

$$\text{(a)} \quad A = \begin{pmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{pmatrix}, \qquad \text{(b)} \quad A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 3 \end{pmatrix},$$

and an eigenvector $u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ to the real eigenvalues λ according to $Au = \lambda u$.

Aufgabe T34: Let $\{u^1, u^2\}$ be a fundamental system of a system of differential equations $u'(x) = A(x)u(x)$ and v be a further solution.

(a) Which dimension has the matrix $A(x)$?

(b) Does $\{u^1, u^2, v\}$ and $\{u^1 + u^2, u^1 - u^2\}$ form a fundamental system, respectively?

Aufgabe T35: Solve the following initial value problem

$$u'(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} u(x)\,, \quad u(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Aufgabe T36: Consider the following linear system of differential equations

$$\begin{aligned} u_1'(x) &= - 2u_2(x), \\ u_2'(x) &= u_1(x) + 2u_2(x). \end{aligned}$$

(a) Determine the (complexed-valued) general solution.

(b) Determine the real-valued solution of the initial value problem

$$u_1(0) = 0, \quad u_2(0) = 1.$$

Tutorial date: Wednesday, June 18.