Universität Karlsruhe (TH)
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56	57	58	59	60	$\sum$

Karlsruhe, 30.06.2008

Student Nr.: ....

Worksheet No.12
Advanced Mathematics II

Exercise 56: (a) Determine the functions $f_1(t)$ and $f_2(t)$ whose Laplace transforms are given by

$$F_1(s) = \frac{2s}{s^4 + 2s^3 + 2s^2 + 2s + 1} \quad \text{und} \quad F_2(s) = \operatorname{arccot}(s-1).$$

(b) Determine the Laplace transform of

$$f_1(x) = x^2 + 3x + 4 + x^2\sin(2x) \quad \text{und} \quad f_2(x) = \begin{cases} \sin(x) & 0 \le x < \pi \\ \cos(x) & x \ge \pi \end{cases}.$$

Exercise 57: (a) Determine the Laplace transform of the following function

$$f(t) = (3 - t^2)\sin t - 3t\cos t.$$

(b) Solve the following initial value problem by means of a Laplace transformation:

$$u^{(IV)}(t) + 2u''(t) + u(t) = \sin(t), \quad u(0) = u'(0) = u''(0) = u'''(0) = 0, \quad t \ge 0.$$

Exercise 58: Solve the initial value problem for an inhomogeneous linear third-order ordinary differential equation

$$xu'''(x) - u''(x) = \frac{1}{6}x^3, \quad x \in [0,\infty), \quad u(0) = 0, \quad u'(0) = 1, \quad u''(0) = 0,$$

by means of a Laplace transformation. Is the solution unique?

Hint: Use differentiation in the original *and* in the image space. In this way you can get a (simpler!) differential equation for the Laplace transform of $u(x)$.

Exercise 59: Solve the following initial value problem by means of a Laplace transformation:

$$u''(t) + 4u'(t) + 3u(t) = \begin{cases} 1, & 0 \le t < 1, \\ 0, & t \ge 1, \end{cases} \quad u(0) = u'(0) = 0.$$

Exercise 60: Solve the following initial value problem

$$u'(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} u(x), \quad u(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad x \ge 0,$$

by means of a Laplace transformation.

Due date: Please hand in your homework on Wednesday, July 9, 09:30.

Tutorial 11
Advanced Mathematics II

Aufgabe T41: Determine the Laplace transforms of the following functions:

(a) $f(t) = 3e^{4t} + 2$, (b) $h(t) = e^{-t}\cos(2t)$, (c) $g(t) = \begin{cases} \sin(\omega t - \varphi), & \text{für } \omega t - \varphi \geq 0, \\ 0, & \text{sonst,} \end{cases}$ mit $\omega, \varphi > 0$.

(d) Determine the Laplace transform of the function

$$f(t) = t^2 \sin^2 t, \quad t \in [0, \infty),$$

by means of twice differentiating in the associated image space. Hint: $\sin^2 t = (1 - \cos 2t)/2$.

Aufgabe T42: Specify which functions $f(t)$ have been Laplace-transformed in the given examples:

$$\text{(a) } F(s) = \frac{1}{s^5} \quad \text{(b) } F(s) = \frac{6}{(s-2)^4} \quad \text{(c) } F(s) = \frac{1}{s(s+1)^2} \quad \text{(d) } F(s) = \frac{2s}{(s+1)^2(s^2+1)}.$$

Aufgabe T43: (a) Solve the initial value problem

$$x''(t) + 2x'(t) + x(t) = 3te^{-t}, \quad x(0) = 4, \quad x'(0) = 2, \quad t \geq 0$$

by means of a Laplace transformation.

(b) Apply a Laplace transformation to the homogeneous linear third-order ordinary differential equation

$$x^3 y'''(x) - 6x^2 y''(x) + 15xy'(x) - 15y(x) = 0, \qquad x \in (0, \infty).$$

Give reasons for whether or not the differential equation can be solved in this way.

Aufgabe T44: Determine the solution of the initial value problem

$$y'(x) = \begin{pmatrix} 4 & 4 \\ -2 & 0 \end{pmatrix} y(x), \quad y(0) = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \quad y(x) \geq 0,$$

by means of a Laplace transformation.

Tutorial date: Wednesday, July 2.