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Karlsruhe, 30.6.2009

Student Nr.: ....

Worksheet No. 11
Advanced Mathematics II

Exercise 51: Consider the functions

$$h(t-a) = \begin{cases} 0, & t < a \\ 1, & t \geq a \end{cases} \qquad \text{and} \qquad f(t) = \begin{cases} t-1, & 1 \leq t < 3, \\ 8-2t, & 3 \leq t < 4, \\ 0, & \text{else.} \end{cases}$$

(a) Rewrite the function f into a form without any case differentiation using the Heaviside step function $h(t-a)$.

(b) Sketch the graph of the function f and determine its Laplace-transform.

Exercise 52: Check if the given parameter integral

$$J(t) = \int_0^1 \arcsin(tx)dx \quad , \quad 0 \leq t < 1 \quad ,$$

is differentiable. Evaluate it by first determining its derivative $J'(t)$ on the open interval $0 < t < 1$. From this result, derive $J(t)$, $0 \leq t < 1$, and determine the value of the integration constant from the value of $J(t)$ at $t = 0$. Can $J(t)$ be smoothly continued at $t = 1$?

Exercise 53: Evaluate the convolution $f * g$ for the functions given by

(a) $f(t) = \sin(t), \qquad g(t) = \begin{cases} 0, & 0 \leq t < 3 \\ 2, & t \geq 3, \end{cases}$

(b) $f(t) = \cosh(t), \qquad g(t) = \sin^2(t).$

Also determine $\mathcal{L}(f * g)$ and compare the result with $\mathcal{L}f \cdot \mathcal{L}g$.

Exercise 54: Given is an initial value problem

$$\begin{aligned} -3x'(t) + x(t) + 2y'(t) + 2y(t) &= 6e^t + 5e^{-2t}, \\ x'(t) - x(t) + y'(t) - y(t) - z'(t) + z(t) &= \frac{3}{2} + \frac{3}{2}e^{-2t}, \\ 2x'(t) - y(t) - z'(t) &= 0, \end{aligned}$$

with conditions

$$x(0) = 2, \quad y(0) = 3, \quad z(0) = 4\,.$$

Determine the first component of the solution, namely the function $x : [0, \infty) \to \mathbb{R}$. Hint: Apply the Laplace-transform.

Exercise 55: A car with a wheelbase of 3 m and equal hubloads of $F = 7200$ N is parked on a 30 m long bridge, 6 m in from the right bearing. The bridge has the special load per unit of length of $q_0 = 14400$ N/m and the bending stiffness of $EJ = 6 \cdot 10^9$ [Nm^2]. Given these characteristics, the displacement w of the bridge satisfies the ordinary differential equation

$$EJ \cdot w''''(x) = q_0 + F\delta(x-21) + F\delta(x-24), \qquad w(0) = w''(0) = w(30) = w''(30) = 0.$$

Determine the function $w(x)$ by means of a Laplace transformation. Hint: You may set $w'(0) = A$ and $w'''(0) = B$, and obtain the constants A and B at the end from the conditions $w(30) = w''(30) = 0$.

Due date: Please hand in your homework on Thursday, July 9, 11:15 a.m.

Tutorial No. 11
Advanced Mathematics II

Exercise T41: Given the function $\Lambda(x) = \begin{cases} x, & 0 \le x < 1 \\ 2-x, & 1 \le x < 2 \\ 0, & \text{else} \end{cases}$,

(a) determine the Laplace-transform $\mathcal{L}\Lambda(s)$,

(b) describe $f(x)$ by means of $\Lambda(x)$ in closed form :

$$f(x) = \begin{cases} 4x, & 0 \le x < 1 \\ -x+5, & 1 \le x < 2 \\ x^2 - 5x + 9, & 2 \le x < 3 \\ -x^2 + 4x, & 3 \le x < 4 \\ 0, & x \ge 4 \end{cases},$$

(c) use this to describe the Laplace-transform $\mathcal{L}f(s)$ by means of $\mathcal{L}\Lambda(s)$.

Exercise T42: Verify the continuity and differentiability of the parameter integral

$$J(t) = \int_0^1 \arctan(tx)\, dx\,.$$

Compute its derivative for $t \in \mathbb{R}$. Does the limit of the derivative exist for $t \to 0$?

Exercise T43: Determine the convolution $f * g$ for the following pair of functions

(a) $f(t) = t^2$; $\quad g(t) = 1 - h(t-1)$, where $h(t) = \begin{cases} 0, & t < 0 \\ 1, & t \ge 0 \end{cases}$.

(b) $f(t) = \sinh t$; $\quad g(t) = \sin 2t$.

Compute the Laplace-transform $\mathcal{L}(f * g)$ and compare the result with $\mathcal{L}f \cdot \mathcal{L}g$.

Exercise T44: Solve the initial value problem

$$\begin{array}{rcrcrcrcr} & & x(t) & - & 2y(t) & + & z(t) & = & -2t, \\ -x'(t) & + & 3y'(t) & - & 2x(t) & + & y(t) & = & 3+t, \\ & & 3z''(t) & - & 5x'(t) & - & 2z(t) & = & 0, \end{array}$$

$$x(0) = 1, \quad y(0) = -1, \quad z(0) = -3, \quad z'(0) = 2$$

by means of Laplace-transform.

Tutorial date: Monday, July 6, 3:45 p.m.–5:15 p.m.