Universität Karlsruhe (TH)
Institut für Algebra und Geometrie
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Karlsruhe, 21.4.2009

Student Nr.: .....

Worksheet No.1
Advanced Mathematics II

Exercise 1: Consider the initial value problem

$$0 = (x+1)y'(x) + 2y(x) - 1, \quad y(0) = \frac{3}{2}.$$

(a) Determine the general solution of the homogeneous differential equation $0 = (x+1)y'(x) + 2y(x)$.

(b) Determine the general solution of the inhomogeneous differential equation and solve the initial value problem.

Exercise 2: Determine the general solution of the following differential equations:

(a) $y'(x) = 2xy(x)$,

(b) $y'(x) = 2y^2(x) + 2 + y^2(x)x + x$,

(c) $xy'(x) = y(x) + xe^{-\frac{y(x)}{x}}$ using the substitution $z(x) = \frac{y(x)}{x}$.

Exercise 3: Determine the type of the differential equation and solve the initial value problem

$$y^3(x) - x^2 + xy^2(x)y'(x) = 0, \qquad y(1) = 1.$$

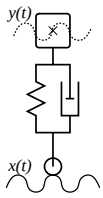
Exercise 4: Determine the type and the solutions of the following initial value problems:

(a) $y'(x) = \dfrac{1}{1-x}y(x) + x - 1, \quad x > 1, \quad y(2) = 0,$

(b) $y'(x) = \sqrt{1 - y^2(x)}, \quad y(0) = \dfrac{1}{2}.$

Exercise 5: On his daily trip to the University, Mr. H. passes various cobblestone pavement paths with his bicycle. He would like to calculate the amplitude of the pavement $x(t)$ from the vertical body movement $y(t)$.

We model the situation as follows: let the weight of Mr. H. be 100 kg, simplify the bycicle to a unicycle, and let the tyre be a parallel connection of spring with 1500 $\frac{N}{m}$ and damper with 500 $\frac{kg}{s}$. The balance of forces leads to the equation in canceled form



$$-my''(t) - b(y'(t) - x'(t)) - c(y(t) - x(t)) = 0$$

where $m = 1$, $b = 5$ and $c = 15$ are unitless constants.

(a) Propose and solve the homogeneous differential equation for $x(t)$. What is the behaviour of $x(t)$ for $t \to \infty$?

(b) Propose now the inhomogeneous differential equation for $x(t)$ for two different pavement paths, where $y_1(t) = \sin(t)$ and $y_2(t) = \sin(2t)$.

(c) Determine the particular solution of the inhomogeneous differential equation using the ansatz

$$x_1(t) = A_1 \sin(t) + B_1 \cos(t) \qquad \text{and} \qquad x_2(t) = A_2 \sin(2t) + B_2 \cos(2t)$$

respectively. Is the assumption of Mr. H. right that the pavement of the second path features a bigger amplitude than the first one for $t \to \infty$?

Due date: Please hand in your homework on Thursday, April 30, 11:15.

Tutorial No.1
Advanced Mathematics II

Exercise T1: Let $I \subset \mathbb{R}$ denote an interval. Give examples for the differential equations for the function $y = y(x)$, $x \in I$ with properties as follow:

	Example
linear	
non-linear	
separable and linear	
separable and non-linear	
autonomous	
homogeneous linear	
inhomogeneous linear	

Exercise T2: Determine the types and the solution of the following initial value problems:

(a) $y'(x) = 2\frac{y(x)}{x}$ with $y(1) = 2$,

(b) $y'(x) = 2\frac{y(x)}{x} + x$ with $y(1) = 2$,

(c) $y'(x) = \frac{y(x)}{x} + \frac{x}{2y(x)}$ with $y(1) = \sqrt{2}$.

Exercise T3: Solve the initial value problem

$$y'(x) + \frac{1}{2}y(x) + xe^x y^3(x) = 0\,,\; y(0) = 1$$

for $x \geq 0$.

Exercise T4: Determine by separation of variablesthe solution of the (non-linear) differential equation of first order

$$u'(x) = \frac{x}{3\sqrt{1+x^2}\,[u(x)]^2}$$

fulfilling the condition $u(0) = 3$.

Tutorial date: Monday, April 27.