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Student Nr.: ....

Worksheet No.2
Advanced Mathematics II

Exercise 6: Let

$$u = \begin{pmatrix} 1 \\ -2 \\ -1 \\ 0 \end{pmatrix}, \qquad v = \begin{pmatrix} 2 \\ 0 \\ 3 \\ -1 \end{pmatrix}, \qquad w = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \qquad z = \begin{pmatrix} 2 \\ 2 \\ 3 \\ -3 \end{pmatrix}$$

be vectors in $\mathbb{R}^4$.

(a) Compute the following linear combinations: $u + v - z$, $2v - (w - z)$, $2u - v + 2w$.

(b) Show that the vectors u, v, w are a basis of the subspace span $\{u, v, w\}$.

(c) What is the dimension of the subspace span $\{u, v, w, z\}$?

Exercise 7: Determine all linear combinations of

(a) $a^{(1)} = (4,1,1)^\top$, $a^{(2)} = (1,2,3)^\top$, $a^{(3)} = (5,6,7)^\top$,

(b) $b^{(1)} = (2,1,1)^\top$, $b^{(2)} = (1,-1,6)^\top$, $b^{(3)} = (5,1,8)^\top$,

(c) $c^{(1)} = (5,1,4)^\top$, $c^{(2)} = (4,1,3)^\top$, $c^{(3)} = (-1,3,-4)^\top$

which describe $x = (3,1,2)^\top$.

Exercise 8: Consider the plane $E : 4x_1 + x_3 + 8 = 0$, the point $P = (2|1|1)$ and the line $H : x(\lambda) = (4,3,-2)^\top + \lambda(3,1,-1)^\top$, $\lambda \in \mathbb{R}$.

(a) Determine a line G through P that is orthogonal to E.

(b) Determine the distance from P to E as well as the point Q in E closest to P.

(c) Determine the point at which the line H intersects E and the point R on H, that is closest to P.

Exercise 9: Let the points $P = (2|1|0)$, $Q = (1|3|-1)$ and $R = (0|2|0)$ be given.

(a) Represent the plane E through the points P, Q and R in both parametric and normal form.

(b) Does the line $G : x(u) = (-2,-7,0)^\top + u(3,2,1)^\top$ intersect the plane E? If so determine the point and angle of intersection.

(c) Compute the orthogonal projection of the direction vector $(3,2,1)^\top$ of the line G onto the normal vector of the plane E. Using this and the intersection point of G and E determine the projection H of the line G onto E.

Exercise 10: $C[0,1]$ denotes the vector space of continuous functions on the closed interval $[0,1]$. Let U be a subspace of $C[0,1]$ spanned by two polynomials $b^{(1)}(x) = 1$ and $b^{(2)}(x) = x - \frac{1}{2}$. We define $y(x) := \sqrt{x} \in C[0,1]$ and the scalar product of two functions by

$$\langle f, g \rangle := \int_0^1 f(x)\overline{g(x)}\, dx \in \mathbb{C}\,.$$

(a) Find a linear combination $c = a_1 b^{(1)} + a_2 b^{(2)} \in U$, such that $c(0) = y(0)$ and $c(1) = y(1)$.

(b) Determine $d \in U$ with smallest distance to y, i.e. the distance vector $e = d - y$ must be orthogonal to $b^{(1)}$ and $b^{(2)}$. Draw the graphs of y and the approximations c and d of it in $[0,1]$ in a figure.
Remark: The Finite Element Method includes the computing of orthogonal approximations like d.

Due date: Please hand in your homework on Thursday, May 7, 11:15.

Tutorial No.2
Advanced Mathematics II

Exercise T5: Let

$$u = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}, \qquad v = \begin{pmatrix} -9 \\ 2 \\ 4 \end{pmatrix}, \qquad w = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$$

be vectors in $\mathbb{R}^3$.

(a) Compute the following linear combinations of these vectors: $u + w$, $v - 3u$, $2u - v + w$.

(b) Show that every pair of vectors in the set $\{u, v, w\}$ are linearly independent.

(c) Are the three vectors also linearly independent as a triple?

Exercise T6: For which $\alpha \in \mathbb{R}$ is $x = (-7, \alpha, 2)^\top$ a linear combination of $a^{(1)} = (1, 2, 4)^\top$, $a^{(2)} = (-2, 1, 2)^\top$ and $a^{(3)} = (3, 1, 2)^\top$? Determine all possible linear combinations of x.

Exercise T7: Let

$$E : x_1 - x_3 = 0 \qquad \text{and} \qquad F : x_1 + 2x_2 + x_3 = 4.$$

be planes in $\mathbb{R}^3$

(a) Find the intersection line G between E and F.

(b) For another straight line H, which lies neither in E nor in F, exist only the following possibilitites:

- it intersects E as well as F in only one point. (Find the angles of intersection.)
- it intersects one of them in only one point, but doesn't intersect the other at all,
- it doesn't intersect any of them.

Construct an example for each of these possibilities and visualize the geometric position of the planes and the straight line.

Exercise T8: Consider the points $P = (2|1| - 4)$, $Q = (-1| - 5| - 1)$ and the plane $E : x_1 + 2x_2 - x_3 = 2$.

(a) Determine a parametric representation of the line G through P and Q, as well as a parametric representation of E.

(b) Compute the point S of intersection of G and E, and show that the line G and the plane E intersect at right angles (i.e. orthogonally). How far is P from E?

Tutorial date: Monday, May 4, 3:45pm-5:15pm