Universität Karlsruhe (TH)
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Karlsruhe, 12.5.2009

Student Nr.: . . . .

Worksheet No. 4
Advanced Mathematics II

Exercise 16: Compute the determinant

$$D := \begin{vmatrix} -3 & 0 & 0 & 7 \\ 6 & 4 & -1 & -3 \\ 0 & -5 & 2 & 2 \\ 3 & -7 & 1 & 0 \end{vmatrix}$$

(a) using the expansion rule across the 1st row,

(b) using the expansion rule across the last column,

(c) via Gaussian elimination.

Exercise 17: Compute the determinant of the matrix A

$$A = \begin{pmatrix} -1 & 2 & 4 & 2 & 1 \\ -2 & -2 & 4 & -1 & 6 \\ 2 & -1 & -5 & -5 & -2 \\ 3 & -3 & -11 & -2 & -4 \\ 2 & -1 & -3 & -7 & -6 \end{pmatrix}.$$

Exercise 18: Given

$$A = \begin{pmatrix} 5 & i-1 & 7 & -4 \\ 5 & \frac{1}{2}(1-i) & 5 & -3 \\ 4 & 0 & 4 & -2 \\ 0 & 0 & 1 & -1 \end{pmatrix} \in \mathbb{C}^{4\times 4} \quad \text{and} \quad B = \begin{pmatrix} 1 & -i-1 & -2 & -2 \\ 2 & -i-1 & -4 & -4 \\ 3 & -5i-5 & -4 & -4 \\ 4 & -7i-7 & -6 & -7 \end{pmatrix} \in \mathbb{C}^{4\times 4}.$$

Determine $\det(A)$ and $\det(B)$, as well as $\det(AB^*)$ and $\det(A^{-1}B)$.

Exercise 19: We consider the question on which conditions a polynomial p of 2nd degree is uniquely defined by $p(a_1) = b_1$, $p(a_2) = b_2$, $p(a_3) = b_3$, i.e. its graph is passing through three points.

(a) Compute the determinant

$$\begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_2 & a_2^2 \\ 1 & a_3 & a_3^2 \end{vmatrix}.$$

When does this determinant have the value 0?

(b) Set up the system of linear equations for the coefficients c_1, c_2, c_3 of the polynomial $p(x) = c_1 + c_2 x + c_3 x^2$, in case this polynomial fulfils the conditions $p(a_j) = b_j$ for $j = 1, 2, 3$.

(c) On which constraints on a_j, b_j, $j = 1, 2, 3$ does the system of linear equations for the coefficients c_j, $j = 1, 2, 3$ have exactly one solution?

Exercise 20: Consider the linear mapping $\psi : \mathbb{R}^3 \to \mathbb{R}^3$ represented by the following matrix

$$A = \begin{pmatrix} -1 & -1 & 2 \\ -2 & -1 & 2 \\ -2 & -2 & 3 \end{pmatrix}$$

with respect to the standard basis. Determine the vector b_3 which fulfils the equation $\psi(b_3) = b_3$. Show that $B = \{b_1, b_2, b_3\}$ is a basis where $b_1 = (1, 1, 1)^\top$, $b_2 = (0, 1, 1)^\top$ and arrange the matrix of ψ with respect to the basis B.

Due date: Please hand in your homework on Wednesday, May 20, 9:30 a.m.

Tutorial No. 4
Advanced Mathematics II

Exercise T13: Evaluate the determinants

$$\text{(a) } D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}, \qquad \text{(b) } D = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{vmatrix}, \qquad \text{(c) } D = \begin{vmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{vmatrix}.$$

Exercise T14: Calculate the determinant of a real-valued 4×4 matrix

$$\begin{vmatrix} 1 & 2 & \pi & 1 \\ 2 & 4 & 1 & 0 \\ 1 & 0 & 0 & 3 \\ 2 & 4 & -1 & 0 \end{vmatrix}$$

(a) using the expansion rule across the 3rd row,

(b) using the expansion rule across the 4th column,

(c) via Gaussian elimination.

Exercise T15:

(a) Compute the determinant of the matrix $A_\alpha \in \mathbb{R}^{4\times 4}$ depending on $\alpha \in \mathbb{R}$, where

$$A_\alpha = \begin{pmatrix} 1 & 1 & 0 & 1 \\ -1 & \alpha & 1 & -1 \\ 0 & 0 & 2 & \alpha \\ 1 & 1 & 0 & \alpha+1 \end{pmatrix}.$$

(b) For which values of $\alpha \in \mathbb{R}$ is the system of linear equations $Ax = b$ solvable if $b = (2, -1, 0, 0)^\top$?

Exercise T16: The linear map $\Phi : \mathbb{R}^2 \to \mathbb{R}^2$ is given by the matrix

$$A = \frac{1}{2}\begin{pmatrix} 7 & 10 \\ -5 & -7 \end{pmatrix}$$

with respect to the standard basis $E = \{e^{(1)}, e^{(2)}\}$. A basis $B = \{b^{(1)}, b^{(2)}\}$ of the vector space $\mathbb{R}^2$ is given also, where $b^{(1)} = (-1, 1)^\top$ and $b^{(2)} = (-3, 2)^\top$.

(a) Give the basis transformation matrix from B to E, i.e. a matrix which transforms the vetors $x = \alpha_1 b^{(1)} + \alpha_2 b^{(2)} = (\alpha_1, \alpha_2)_B^\top$ with respect to the basis B into the representation $x = \beta_1 e^{(1)} + \beta_2 e^{(2)} = (\beta_1, \beta_2)_E^\top$.

(b) Give the basis transformation matrix from E to B, i.e. a matrix which transforms the vectors with respect to the standard basis E into the vectors with respect to the basis B.

(c) Give the matrix of the map Φ with respect to the basis B. What is Φ?

Tutorial date: Monday, May 18, 3:45 p.m.–5:15 p.m.