Universität Karlsruhe (TH)
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26	27	28	29	30	$\sum$

Karlsruhe, 26.5.2009

Student Nr.: ....

Worksheet No. 6
Advanced Mathematics II

Exercise 26: Let the following differential equation for $x > 0$ be given:

$$y'''(x) - \frac{2}{x}y''(x) + \frac{5}{x^2}y'(x) - \frac{5}{x^3}y(x) = 0$$

Check, if the following functions are solutions of this differential equation:

(a) $y_1(x) = \sin(x^2)$,

(b) $y_2(x) = x$,

(c) $y_3(x) = \exp(\frac{2}{x})$,

(d) $y_4(x) = x^2 \cos(\ln(x))$.

Exercise 27: Determine the general real-valued solution of the homogeneous differential equation

$$y'''(x) + 2y''(x) + 2y'(x) + y(x) = 0$$

using the exponential ansatz $y(x) = e^{\lambda x}$, $\lambda \in \mathbb{C}$.

Exercise 28: Solve the initial value problems:

(a) $y''(x) - 2y'(x) - 3y(x) = 0\,,\ x \in \mathbb{R}\,,\quad y(0) = 0,\ y'(0) = -4;$

(b) $y'''(x) - 4y'(x) = 0,\ x \in \mathbb{R}\,,\quad y(0) = 0\,,\ y'(0) = 0,\ y''(0) = 2.$

Exercise 29: Determine the real-valued general solution of the differential equation

$$x^4u''''(x) + 6x^3u'''(x) - 2xu'(x) + 20u(x) = 0, \quad x > 0.$$

Exercise 30: Show that $u(x) = e^{x^2}$ solves the homogeneous differential equation

$$u''(x) - 2xu'(x) - 2u(x) = 0, \quad x \in (0, \infty)\ .$$

Determine a second non-trivial solution by means of the method of reduction of the order.

Due date: Please hand in your homework on Thursday, June 4, 11:15 a.m.

Tutorial No. 6
Advanced Mathematics II

Exercise T21: Determine the general solution of the linear homogeneous differential equations with constant coefficients:

(a) $y'''(x) - 3y''(x) - y'(x) + 3y(x) = 0\,,\ x \in \mathbb{R}\,,$

(b) $y'''(x) + 7y''(x) + 19y'(x) + 13y(x) = 0\,,\ x \in \mathbb{R}\,.$

Exercise T22: Consider the homogeneous Euler differential equation

$$2x^3u'''(x) + Bx^2u''(x) + xu'(x) - 10u(x) = 0, \quad x > 0.$$

(a) Determine $B \in \mathbb{R}$ so that $u_1(x) = x^{\frac{5}{2}}$ is a solution of the differential equation.

(b) Determine the general solution of the differential equation for the computed constant B from part (a).

Exercise T23: Determine the real general solution of the differential equation

$$u'''(x) + 3u''(x) + 9u'(x) - 13u(x) = 0, \quad x \in \mathbb{R}.$$

Show by means of the general solution that every initial value problem $u(0) = a$, $u'(0) = b$, $u''(0) = c$ has a unique solution.

Exercise T24: Show that $y(x) = x$ fulfils the differential equation

$$(1 + x^2)y''(x) - 2xy'(x) + 2y(x) = 0\,,\ x \in \mathbb{R}$$

and determine other solution by means of the method of reduction of the order.

Tutorial date: Monday, June 1, 3:45 p.m.–5:15 p.m.