Universität Karlsruhe (TH)
Institut für Algebra und Geometrie
PD Dr. F. Hettlich
Dipl.-Math.techn. A. Schkarbanenko

31	32	33	34	35	$\sum$

Karlsruhe, 2.6.2009

Student Nr.: ....

Worksheet No. 7
Advanced Mathematics II

Exercise 31: Determine the general real-valued solution of the differential equation

$$0 = x^4 y^{(4)}(x) + 2x^3 y'''(x) + 3x^2 y''(x) - 3xy'(x) + 4y(x)\,, \quad x > 0\,.$$

Hint: The characteristic polynomial is divisible by $q(\lambda) := \lambda^2 - 2\lambda + 2$.

Exercise 32: Solve the initial value problem

$$\begin{aligned} 0 &= (2x^2 + x)y''(x) - (4x^2 - 2)y'(x) - (4x + 4)y(x)\,, \quad x \geq 1\,, \\ & y(1) = 3\,,\ y'(1) = 0\,. \end{aligned}$$

Hint: The function $y_1(x) = \frac{1}{x}$ satisfies this differential equation.

Exercise 33: Solve the initial value problem of the following linear system

$$\begin{aligned} u'(x) &= v(x)\,, & u(0) &= 2\,, \\ v'(x) &= w(x)\,, & v(0) &= 2\,, \\ w'(x) &= 4u(x) - 4v(x) + w(x)\,, & w(0) &= -3\,. \end{aligned}$$

Exercise 34: Find the solution of the initial value problem for the complex-valued liner system

$$u'(x) = \begin{pmatrix} 1 - \frac{1}{7}i & \frac{6}{7} \\ -\frac{65}{42} + i & -1 + \frac{5}{7}i \end{pmatrix} u(x), \quad x \in [0, \infty), \quad u(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

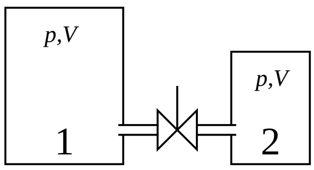
Exercise 35: *Two pressure tanks with different capacities V_1 and V_2 are linked by a pipe which is closed by a valve. Before opening the stop valve at time $t = 0$ the air in the tanks have two different pressures $p_1(0)$ and $p_2(0)$. By the ideal gas law $pV = nRT$ (n amount of substance, R gas constant, T temperature) assuming isothermal balancing we achieve the relation $\dot{p}V = \dot{n}RT$ and finally with flow resistance W of the pipe, $\dot{n} = Wp$ and the notation $a_{1,2} := \frac{RT}{WV_{1,2}}$ the following system for the model*

$$\begin{pmatrix} \dot{p}_1(t) \\ \dot{p}_2(t) \end{pmatrix} = \begin{pmatrix} -a_1 & a_1 \\ a_2 & -a_2 \end{pmatrix} \begin{pmatrix} p_1(t) \\ p_2(t) \end{pmatrix}.$$

Let $p_1(0) = 1$ bar, $p_2(0) = 9$ bar, $a_1 = 1$ bar/s and $a_2 = 3$ bar/s.

(a) In which tank and when is the pressure two bar?

(b) Which pressure will be obtained when the system is completely balanced?



Due date: Please hand in your homework on Thursday, June 11, 11:15 a.m.

Tutorial No. 7
Advanced Mathematics II

Exercise T25: Consider the following linear system of differential equations

$$\begin{array}{rcrcl} u_1'(x) & = & & - & 2u_2(x), \\ u_2'(x) & = & u_1(x) & + & 2u_2(x). \end{array}$$

(a) Determine the (complexed-valued) general solution.

(b) Determine the real-valued solution of the initial value problem

$$u_1(0) = 0, \quad u_2(0) = 1.$$

Exercise T26: Solve the following initial value problem

$$u'(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} u(x)\,, \quad u(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Exercise T27: Determine the general solution of the system

$$x'(t) = Ax(t) = \begin{pmatrix} -3 & 1 \\ -1 & -1 \end{pmatrix} x(t).$$

Show therefore:

(a) $\lambda = -2$ is a zero of the characteristic polynomial of A with multiplicity two and $v^{(1)} = (1,1)^\top$ is the corresponding eigenvector.

(b) The ansatz $x(t) = e^{\lambda t} v^{(2)} + t e^{\lambda t} v^{(1)}$ yields the equation $(A - \lambda E)v^{(2)} = v^{(1)}$. One solution is $v^{(2)} = \binom{0}{1}$.

(c) The functions $x^{(1)}(t) = e^{\lambda t} v^{(1)}$ and $x^{(2)}(t) = e^{\lambda t} v^{(2)} + t x^{(1)}(t)$ provide the fundamental system.

Exercise T28: Given the following differential equation for y in $x \in \mathbb{R}$:

$$y'''(x) + y''(x) - 4y'(x) - 4y(x) = 0.$$

(a) Find the general solution $y(x)$.

(b) Let $u_1(x) = y(x)$, $u_2(x) = y'(x)$ and $u_3(x) = y''(x)$. Represent the derivative u_k', $k = 1,2,3$ by the functions u_l, $l = 1,2,3$ and use this to set up a linear system for $u(x)$.

(c) Determine the general solution of the linear system using your results from part (a).

Tutorial date: Monday, June 8, 3:45 p.m.–5:15 p.m.