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Karlsruhe, 9.6.2009

Student Nr.: ..................

# Worksheet No. 8
# Advanced Mathematics II

**Exercise 36:** Specify the roots of the characteristic polynomial and an ansatz for a particular solution by special right-hand sides of the following differential equations for $y = y(x)$:

(a) $y'' + y = x \sin x$
(b) $y''' - 4y'' - 2y' + 20y = x^2 e^x$
(c) $y''' + 6y'' + 12y' + 8y = xe^{-2x}$
(d) $y''' + y'' - 6y' = xe^{2x} + 2e^{-3x}$
(e) $y^{(4)} + 4y''' + 6y'' + 4y' + 5y = -8\cos x - 8 \sin x$
(f) $y^{(5)} + y^{(4)} - 4y''' - 16y'' - 20y' - 12y = e^{-3x}$

Hint: A solution of (e) is $y(x) = x\cos(x)$ and one solution of the homogeneous differential equation in (f) is $y(x) = x\sin(x)e^{-x}$.

**Exercise 37:** Determine the general solution of the inhomogeneous differential equation

$$y'''(x) + 3y''(x) + 3y'(x) + y(x) = x + 6e^{-x}\ ,\ x \in \mathbb{R}.$$

Use the method of undetermined coefficients to determine a particular solution.

**Exercise 38:** Consider the ordinary differential equation

$$x^2 y''(x) - 2xy'(x) + 2y(x) = x^3 \ln x, \quad x > 0.$$

The associated homogeneous differential equation has a solution of the form $y(x) = Ax + B$.

(a) Determine the general solution of the homogeneous problem using the method of reduction of the order.

(b) Determine a particular solution of the inhomogeneous problem by means of variation of parameters.

(c) Solve the initial value problem for the inhomogeneous ordinary differential equation with $y(1) = y'(1) = 1$.

**Exercise 39:** Consider the ordinary differential equation

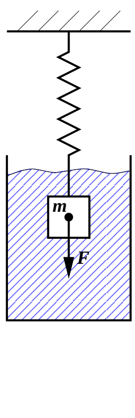
$$-15u(x) + 3xu'(x) + x^2u''(x) = 8x^{-3}, \qquad x > 0.$$

(a) Find a real-valued fundamental system of the associated homogeneous differential equation.

(b) Find a particular solution by the method of variation of parameters. Determine the general solution of the inhomogeneous problem.

**Exercise 40:** A mass $m$ of 5 kg streches a spring about 0.1 m. This system is placed in a viscous fluid. Due to the fluid a braking force of 2 N acts on the mass if the velocity is 0.04 m/s. Additionally an exterior force $F(t) = 2\,\cos(\omega t)\,\mathrm{N}$, $t > 0$, $\omega \in \mathbb{R}$ acts on the mass. For the acceleration of gravity we can assume $g = 10\,\mathrm{m/s}^2$.

(a) Set up from the balance of forces for spring force $F_F(t) = -Du(t)$, damping $F_D(t) = -\sigma u'(t)$, inertia $F_T(t) = -mu''(t)$ and the exterior force $F(t)$ the appropriate differential equation and find the general solution.




(b) A summand in the solution, one calls it *stationary solution*, renders the behaviour of the system for large time. It is independent from initial conditions.

Write the stationary solution as

$$A(\omega)\,\cos(\omega\,t - \delta),$$

and find $\omega$ for which the amplitude $A(\omega)$ is maximal.

**Due date:** Please hand in your homework on Thursday, June 18, 11:15 a.m.

# Tutorial No. 8
# Advanced Mathematics II

**Exercise T29:** Determine the characteristic polynomial for the inhomogeneous third-order ordinary differential equation

$$y''' + y'' + 4y' + 4y = f(x)\,.$$

Determine ansätze which reflect the structure of the right-hand side (method of undefined coefficients) for:

$$\begin{array}{lll} f_1(x) = (1+x)e^{2x} & f_2(x) = e^{-x} & f_3(x) = \sin(2x) \\ f_4(x) = x^2\cos(2x) & f_5(x) = xe^{-x}\sin(2x) & f_6(x) = (2+x)\sin(2x) \end{array}$$

**Exercise T30:** Solve the differential equation by the method of undetermined coefficients:

$$y'''(x) - 3y''(x) + 4y(x) = (x^2 + 4x + 2)e^{3x} + e^{2x}\,,\ x \in \mathbb{R}$$

**Exercise T31:** For the inhomogeneous linear second-order ordinary differential equation

$$x^2 y''(x) - \frac{3}{2}xy'(x) + y(x) = x^3\,, \quad x > 0,$$

determine

(a) the general solution of the homogeneous differential equation by means of reduction of the order. Use the fact that $y_1(x) = x^2$ solves the homogeneous problem.

(b) a particular solution and the general solution of the inhomogeneous differential equation by means of variation of the constants.

(c) a solution of the initial value problem with $y(1) = \frac{17}{5}$ and $y'(1) = \frac{21}{5}$.

**Exercise T32:** Determine the general solution of the following differential equation

$$x^2 y''(x) + xy'(x) - 4y(x) = 1 + x^2, \quad x > 0.$$

**Tutorial date:** Monday, June 15, 3:45 p.m.–5:15 p.m.