Universität Karlsruhe (TH)
Institut für Algebra und Geometrie
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Karlsruhe, 16.6.2009

Student Nr.: ....

Worksheet No. 9
Advanced Mathematics II

Exercise 41: Use the power series method to determine the solution of following initial value problem for the Tshebysheff differential equation with index $n \in \mathbb{N}_0$:

$$(1-x^2)u''(x) - xu'(x) + n^2 u(x) = 0, \quad x \in (-1,1), \quad u(0) = 1, \quad u'(0) = 0.$$

Show that

(a) all coefficients of odd powers vanish,

(b) in case of even n the series truncates after the summand including x^n and

(c) in case of odd n the solution's radius of convergence is 1.

Exercise 42: Find the solution of the initial value problem

$$y'' - 2xy' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

using the power series method and determine the radius of convergence.

Exercise 43: The solution of the differential equation

$$y''(x) - x\,y(x) = 0$$

can be expanded as a power series with center of expansion $x_0 = 1$. Give the recursion formula for the coefficients depending on $y(1)$ and $y'(1)$ and compute the first four coefficients.

Exercise 44: Determine a solution of the initial value problem

$$(x^2 + 2x + 2)y''(x) + 2(x+1)y'(x) - 2y(x) = 0, \quad y(-1) = 1, \quad y'(-1) = 0$$

applying the power series method.

Exercise 45: Use the generalized power series method, i.e., employ the ansatz $y(x) = \sum_{k=0}^{\infty} a_k x^{k+\lambda}$ to determine the general solution of the homogeneous differential equation

$$x^2 y''(x) + x^2 y'(x) - 2y(x) = 0.$$

Due date: Please hand in your homework on Thursday, June 25, 11:15 a.m.

Tutorial No. 9
Advanced Mathematics II

Exercise T33: Solve the initial value problem

$$xu''(x) + 4u'(x) + 3u(x) = 3, \quad u(0) = 2,$$

using the power series method. For which $x \in \mathbb{R}$ does the series converge absolutely? You don't have to give the solution in an explicit form.

Exercise T34: Solve the differential equation

$$(2 + x)y''(x) + y'(x) = 1$$

by means of an ansatz in power series form with center of expansion $x_0 = 0$ and determine the radius of convergence.

Exercise T35: Solve the initial value problem

$$(2x - x^2)y''(x) + (1 - x)y'(x) = 0, \quad y(1) = 1, \quad y'(1) = 0$$

with the power series method.

Exercise T36: Determine the general solution of the differential equation

$$x^2y''(x) + x^3y'(x) - 6y(x) = 0\,.$$

Use the generalized power series representation $y(x) = \sum_{k=0}^{\infty} a_k x^{k+\lambda}$.

Tutorial date: Monday, June 22, 3:45 p.m.–5:15 p.m.