(d) <u>Length invariance:</u> Show that $\|R_\alpha x\| = \|x\|$. (Skip $\|R_\alpha y\| = \|y\|$).

1) $\|x\| = \sqrt{x_1^2+x_2^2+x_3^2} = \sqrt{1+1} = \sqrt{2}$

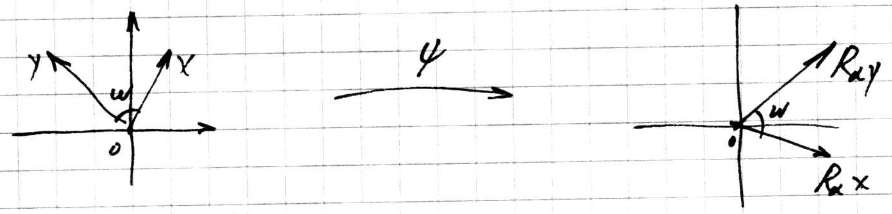
2) $$\|R_\alpha x\| = \left\| \begin{pmatrix}1&0&0\\0&\cos\alpha&-\sin\alpha\\0&\sin\alpha&\cos\alpha\end{pmatrix}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}\right\| = \left\|\begin{pmatrix}1\\-\sin\alpha\\\cos\alpha\end{pmatrix}\right\| = \sqrt{1+\sin^2\alpha+\cos^2\alpha} = \sqrt{2}.$$

Thus the rotation map (represented by R_α) keeps the length of x invariant.

For general case see lecture notes, page 23.

<u>Angle invariance:</u> Show that $\langle x,y\rangle = \langle R_\alpha x, R_\alpha y\rangle$.

$$\cos w = \frac{\langle x,y\rangle}{\cancel{\|x\|\cdot\|y\|}} \overset{!}{=} \frac{\langle R_\alpha x, R_\alpha y\rangle}{\cancel{\|R_\alpha x\|\cdot\|R_\alpha y\|}} \quad \Big|\cdot \|x\|\cdot\|y\|$$



$$\langle x,y\rangle = \left\langle \begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}1\\-1\\0\end{pmatrix}\right\rangle = \underline{\underline{1.}}$$

$$\langle R_\alpha x, R_\alpha y\rangle = \left\langle \begin{pmatrix}1&0&0\\0&\cos&-\sin\\0&\sin&\cos\end{pmatrix}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}1&0&0\\0&\cos&-\sin\\0&\sin&\cos\end{pmatrix}\begin{pmatrix}1\\-1\\0\end{pmatrix}\right\rangle = \left\langle\begin{pmatrix}1\\-\sin\\\cos\end{pmatrix},\begin{pmatrix}1\\-\cos\\-\sin\end{pmatrix}\right\rangle =$$

$$= 1 + \underbrace{\sin\alpha\cos\alpha - \sin\alpha\cdot\cos\alpha}_{=0} = \underline{\underline{1}}$$

So this mapping keeps the angle of intersection between x and y invariant.