4. Aufgabe

(a) $f, g: \mathbb{R}^3 \to \mathbb{R}^2$ mit $f: (x,y,z) \mapsto (3y, 6)^T$, $g: (x,y,z) \mapsto (e^{xy}, z^2)^T$.
Berechnen Sie $(g^T f)'(x,y,z)$. (1. Produktregel)

$$(g^T f)'(x,y,z) = g^T(x,y,z) \cdot f'(x,y,z) + f^T(x,y,z) \cdot g'(x,y,z)$$

$$= (e^{xy}, z^2) \cdot \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial y} \end{pmatrix} + (3y, 6) \cdot \begin{pmatrix} \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y} & \frac{\partial g_1}{\partial z} \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} & \frac{\partial g_2}{\partial z} \end{pmatrix}$$

$$= (e^{xy}, z^2) \cdot \begin{pmatrix} 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix} + (3y, 6) \begin{pmatrix} ye^{xy} & xe^{xy} & 0 \\ 0 & 0 & 2z \end{pmatrix}$$

$$= (0, 3e^{xy}, 0) + (3y^2 e^{xy}, 3xye^{xy}, 12z)$$

$$= (3y^2 e^{xy}, 3e^{xy}(1+xy), 12z).$$

Bem.: $(g^T f)'(x,y,z) : \mathbb{R}^3 \to \mathbb{R}^{1\times 3}$.

1. Priorität!

(b) $f: \mathbb{R}^3 \to \mathbb{R}^2$ wie in (a).
$g: \mathbb{R}^3 \to \mathbb{R}$ mit $g: (x,y,z) \mapsto e^{xy}$
Berechnen Sie $(gf)'(x,y,z)$. (2. Produktregel)

$$(gf)'(x) = g(x,y,z) \cdot f'(x,y,z) + f(x,y,z) \cdot g'(x,y,z)$$

$$\overset{(a)}{=} e^{xy} \begin{pmatrix} 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 3y \\ 6 \end{pmatrix} \cdot (ye^{xy}, xe^{xy}, 0)$$

$$= \begin{pmatrix} 0 & 3e^{xy} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 3y^2 e^{xy} & 3xye^{xy} & 0 \\ 6ye^{xy}, & 6xe^{xy} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 3y^2 e^{xy} & 3e^{xy}(1+xy) & 0 \\ 6ye^{xy} & 6x e^{xy} & 0 \end{pmatrix} = 3e^{xy} \begin{pmatrix} y^2 & (1+xy) & 0 \\ 2 & 2x & 0 \end{pmatrix}.$$

Bem.: $(gf')(x,y,z) : \mathbb{R}^3 \to \mathbb{R}^{2\times 3} \leftarrow \mathbb{R}^3 : f'(x,y,z)$