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Karlsruhe, 13 April 2010

11	12	13	14	15	$\sum$

Student Nr.: ....

Worksheet No.3
Advanced Mathematics II

Exercise 11: Consider the following two subsets of $\mathbb{R}^3$: E is given by $E := \{x \in \mathbb{R}^3 : x_1 - x_3 = 0\}$ and F is a plane that contains the points $A = (4|0|0)$, $B = (3|0|1)$ and $C = (2|1|0)$.

(a) Find the parametric form of the sets E and F.

(b) Determine the intersection $G = E \cap F$.

(c) Which one of the sets G, E, F is a subspace of $\mathbb{R}^3$? Justify your answer.

Exercise 12: Let the line G and the set E in $\mathbb{R}^4$ be given by $G: \; x = (0,1,0,-2)^\top + \lambda\,(2,2,1,0)^\top \;, \; \lambda \in \mathbb{R}\,, \quad E: \; 3x_1 + 4x_4 = 4\,.$

(a) Determine the intersection of G and E.

(b) Find the set F of the type $a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = c$ that contains the point $(1|0|0|0)$ and is perpendicular to G.

(c) Determine all the points on G that have the same distance to the set E as to the set F from part (b).

Exercise 13: Consider the plane $E: \; x(s,t) = (3,1,0)^\top + s(1,-1,2)^\top + t(0,0,2)^\top$ and the two lines $G: \; x(u) = (4,1,0)^\top + u(0,1,0)^\top$ and $H: \; x(v) = (3,3,3)^\top + v(2,-2,1)^\top$.

(a) For each line determine its intersection with E and the distance from E.

(b) Compute the orthogonal projections of G and H onto E.

Exercise 14: Let the line $G: x(s) = (5,1,-1)^\top + s(4,0,-3)^\top,\; s \in \mathbb{R}$, and the two points $P = (2|0|2)$ and $Q = (0|2|2)$ be given.

(a) Determine a parametric representation of the line H that passes through P and Q.

(b) Determine the point R on G such that the plane E_1 passing through P, Q and R is parallel to the plane E_2 given by the equation $2x_1 + 2x_2 - 3x_3 = 0$. How far apart are the two planes?

(c) At what angle does G intersect the two planes?

Exercise 15: Determine matrices corresponding to the following linear maps:

(a) $f: \mathbb{R}^3 \to \mathbb{R}^3, \quad f(x_1,x_2,x_3) = \begin{pmatrix} 3x_3 + x_1 + x_2 \\ 2x_2 + x_3 - x_1 \\ 2x_1 - 3x_2 - x_3 \end{pmatrix}$

(b) Let P_3 be the space of polynomials of degree up to 3, and let $g: P_3 \to P_3$ map each polynomial onto its derivative. The matrix representation should be with respect to the basis of monoms $\{1, x, x^2, x^3\}$.

Due date: Please hand in your homework until Thursday, 06 May, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

Tutorial No.3
Advanced Mathematics II

Exercise T7: Consider the plane $E : 4x_1 + x_3 + 8 = 0$, the point $P = (2|1|1)$ and the line $H : x(\lambda) = (4, 3, -2)^\top + \lambda(3, 1, -1)^\top$, $\lambda \in \mathbb{R}$.

(a) Determine a line G through P that is orthogonal to E.

(b) Determine the distance from P to E as well as the point Q in E closest to P.

(c) Determine the point at which the line H intersects E and the point R on H, that is closest to P.

Exercise T8: Let

$$E : x_1 - x_3 = 0 \qquad \text{and} \qquad F : x_1 + 2x_2 + x_3 = 4.$$

be planes in $\mathbb{R}^3$

(a) Find the intersection line G between E and F.

(b) For another straight line H, which lies neither in E nor in F, exist only the following possibilitites:

- it intersects E in only one point and also F in only one point. (Find the angles of intersection.)
- it intersects one of them in only one point, but doesn't intersect the other at all,
- it doesn't intersect any of them.

Construct an example for each of these possibilities and visualize the geometric position of the planes and the straight line.

Exercise T9: Let the points $P = (2|1|0)$, $Q = (1|3|-1)$ and $R = (0|2|0)$ be given.

(a) Represent the plane E through the points P, Q and R in both parametric and normal form.

(b) Does the line $G : x(u) = (-2, -7, 0)^\top + u(3, 2, 1)^\top$ intersect the plane E? If so determine the point and angle of intersection.

(c) Compute the orthogonal projection of the direction vector $(3, 2, 1)^\top$ of the line G onto the normal vector of the plane E. Using this and the intersection point of G and E determine the projection H of the line G onto E.

For detailed information regarding this course please check the page
`http://www.math.kit.edu/iag1/lehre/am22010s/en`

Tutorial date: Friday 30 April