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Karlsruhe, 11 May 2010

21	22	23	24	25	Σ

Student Nr.: ....

Worksheet No.5
Advanced Mathematics II

Exercise 21: Given $A = \begin{pmatrix} 1 & 4 & 3 & 0 \\ 4 & 7 & 6 & 0 \\ 7 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ 9 \\ 11 \\ 0 \end{pmatrix}$,

(a) compute all solutions x_h of the homogeneous linear system $Ax = 0$. These solutions form a subspace of $\mathbb{R}^4$. Determine a basis of this subspace.

(b) Show: $x_p = (2, 1, -1, -2)^\top$ is a solution of the inhomogeneous linear system $Ax = b$. Determine the set of all solutions of $Ax = b$ by using part (a).

Exercise 22: Compute the following determinants:

$$\text{(a) } D = \begin{vmatrix} 3 & \frac{3}{7} & 2 & \pi \\ 0 & 1 & a & 4 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{vmatrix}, \qquad \text{(b) } D = \begin{vmatrix} 2 & 4 & 2 & -1 \\ 2 & 3 & 0 & 5 \\ 2 & 1 & 2 & 3 \\ 1 & 2 & 0 & 2 \end{vmatrix}, \qquad \text{(c) } D = \begin{vmatrix} 3 & 2 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ -1 & 2 & 0 & 3 \\ 6 & 2 & 3 & 1 \end{vmatrix}.$$

Exercise 23: Compute the determinant of the matrix $C := (AB)^{-1}$ with

$$A := \begin{pmatrix} 1 & 2 & 3 & -2 & 2 \\ 3 & -1 & 4 & 2 & 0 \\ 8 & -3 & -2 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } B := \begin{pmatrix} -1 & 3 & -2 & 2 & -1 \\ 0 & 2 & -3 & -3 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 2 & 5 \\ 0 & 2 & 0 & 0 & 6 \end{pmatrix}.$$

Exercise 24: Let

$$A = \begin{pmatrix} 1 & \alpha & -2 \\ 2 & -1 & \alpha - 1 \\ -1 & \alpha + 1 & 3 \end{pmatrix} \in \mathbb{R}^{3\times 3}.$$

Calculate the determinant of A. For which $\alpha \in \mathbb{R}$ is A invertible? Calculate A^{-1} for $\alpha = -1$.

Exercise 25: The linear map $\Phi : \mathbb{R}^2 \to \mathbb{R}^2$ is given by the matrix

$$A = \frac{1}{2} \begin{pmatrix} 7 & 10 \\ -5 & -7 \end{pmatrix}$$

with respect to the standard basis $E = \{e^{(1)}, e^{(2)}\}$. Another basis $B = \{b^{(1)}, b^{(2)}\}$ of the vector space $\mathbb{R}^2$ is given by $b^{(1)} = (-1, 1)^\top$ and $b^{(2)} = (-3, 2)^\top$.

(a) Determine the basis transformation matrix from B to E, i.e. a matrix which transforms the vectors $x = \alpha_1 b^{(1)} + \alpha_2 b^{(2)} = (\alpha_1, \alpha_2)_B^\top$ with respect to the basis B into the representation $x = \beta_1 e^{(1)} + \beta_2 e^{(2)} = (\beta_1, \beta_2)_E^\top$.

(b) Determine the basis transformation matrix from E to B, i.e. a matrix which transforms the vectors with respect to the standard basis E into the vectors with respect to the basis B.

(c) Determine the matrix of the map Φ with respect to the basis B. Describe Φ geometrically.

Due date: Please hand in your homework until Thursday, 20 May, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

Tutorial No.5
Advanced Mathematics II

Exercise T13: Calculate the determinant of a real-valued 4×4 matrix

$$\begin{vmatrix} 1 & 2 & \pi & 1 \\ 2 & 4 & 1 & 0 \\ 1 & 0 & 0 & 3 \\ 2 & 4 & -1 & 0 \end{vmatrix}$$

(a) using the expansion rule across the 3rd row,

(b) using the expansion rule across the 4th column,

(c) via Gaussian elimination.

Exercise T14: Given

$$A = \begin{pmatrix} 5 & i-1 & 7 & -4 \\ 5 & \frac{1}{2}(1-i) & 5 & -3 \\ 4 & 0 & 4 & -2 \\ 0 & 0 & 1 & -1 \end{pmatrix} \in \mathbb{C}^{4\times 4} \quad \text{and} \quad B = \begin{pmatrix} 1 & -i-1 & -2 & -2 \\ 2 & -i-1 & -4 & -4 \\ 3 & -5i-5 & -4 & -4 \\ 4 & -7i-7 & -6 & -7 \end{pmatrix} \in \mathbb{C}^{4\times 4}.$$

Determine $\det(A)$ and $\det(B)$, as well as $\det(AB^*)$ and $\det(A^{-1}B)$.

Exercise T15: Compute the determinant of the matrix

$$A = \begin{pmatrix} 3 & \alpha-1 & -\alpha+3 \\ 0 & \alpha+3 & -4 \\ 0 & 2 & \alpha-3 \end{pmatrix} \in \mathbb{R}^{3\times 3}.$$

For which α is the linear system of equations $A_\alpha x = b$ for $b = (9, 4, -2)^\top$ solvable?

Tutorial date: Friday 14 May