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Karlsruhe, 18 May 2010

26	27	28	29	30	Σ

Student Nr.: ....

Worksheet No.6
Advanced Mathematics II

Exercise 26: Determine all eigenvalues of the square matrices

$$\text{(a)}\quad A = \begin{pmatrix} 2 & 7 & -2 \\ -1 & -1 & -1 \\ -1 & -5 & 3 \end{pmatrix}, \qquad \text{(b)}\quad B = \frac{1}{2}\begin{pmatrix} 1 & -5 & 5 \\ 5 & -9 & 5 \\ 5 & -5 & 1 \end{pmatrix},$$

and an eigenvector to every real eigenvalue λ.

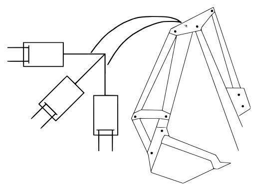
Exercise 27: Consider the matrix $A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 5 & 6 \\ 0 & -3 & -4 \end{pmatrix}$.

(a) Compute the eigenvalues of A and a basis $\{v^1, v^2, v^3\}$ of $\mathbb{R}^3$ which consists only of eigenvectors.

(b) Let P be the matrix with columns v^1, v^2 und v^3. Show that the matrix $D := P^{-1}AP$ is of the form $\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$ where λ_1, λ_2, λ_3 are the eigenvalues of A.

Exercise 28: *By a measurement rosette (Dehnmessstreifen-Rosette) on a surface the extension state of a hydraulic shovel may be determined in the form of a strain tensor with respect to the x_1, x_2, x_3 coordinate system. We are interested in the principle strains (Hauptdehnungen) as they determine the maximal forces in the corresponding principle strain axes in isotropic materials.*

Determine for the given tensor



$$\varepsilon = \frac{1}{50}\begin{pmatrix} 142 & -144 & 0 \\ -144 & 58 & 0 \\ 0 & 0 & -125 \end{pmatrix}$$

the principle strains (i.e. the eigenvalues of ε) and compute for each principle strain a normed eigenvector as corresponding principle strain axis. Determine the angles between the principle strain axes, discuss whether they form a basis of $\mathbb{R}^3$ and determine their angle to the standard basis.

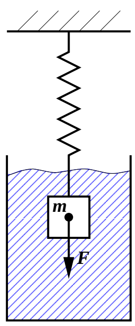
Exercise 29: Determine the general real solution of the linear homogeneous differential equations with constant coefficients:

(a) $y'''(x) - 3y''(x) - y'(x) + 3y(x) = 0\,,\ x \in \mathbb{R}\,,$

(b) $y'''(x) + 7y''(x) + 19y'(x) + 13y(x) = 0\,,\ x \in \mathbb{R}\,,$

(c) $y^{(4)}(x) - 7y'''(x) + 18y''(x) - 20y'(x) + 8y(x) = 0\,,\ x \in \mathbb{R}\,.$

Exercise 30:



A mass m of 5 kg stretches a spring about 0.1 m. This system is placed in a viscous fluid. Due to the fluid a braking force of 2 N acts on the mass if the velocity is 0.04 m/s. For the acceleration of gravity we can assume $g = 10\,\text{m/s}^2$.

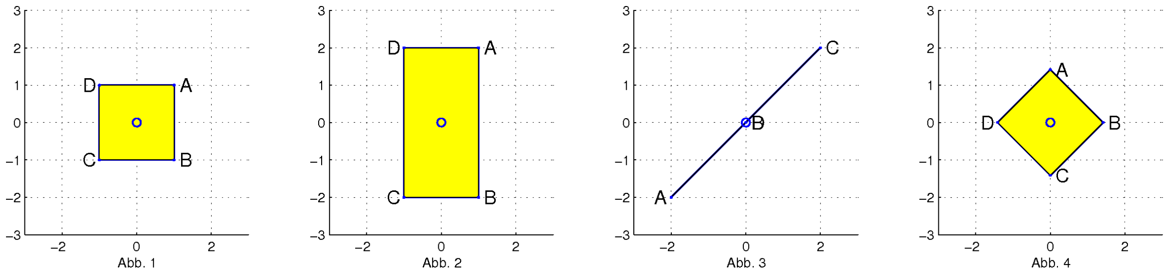
Set up from the balance of forces for spring force $F_F(t) = -Du(t)$, damping $F_D(t) = -\sigma u'(t)$ and inertia $F_T(t) = -mu''(t)$ the appropriate differential equation and find the general (real) solution.

The mass is released 1 m from its position of rest. Compute the solution of this initial value problem.

Due date: Please hand in your homework until Thursday, 27 May, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

Tutorial No.6
Advanced Mathematics II

Exercise T16: Figure 1 shows a square around the origin with the vertices A, B, C, D. The following figures show the images of the square obtained by applying different linear mappings $\Phi_{1,2,3} : \mathbb{R}^2 \to \mathbb{R}^2$:



Determine the eigenvalues of the linear mappings. Sketch the eigenvectors if possible.

Exercise T17: Determine all eigenvalues of the square matrices

$$\text{(a)} \quad A = \begin{pmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{pmatrix}, \qquad \text{(b)} \quad B = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 3 \end{pmatrix},$$

and an eigenvector to every real eigenvalue λ.

Exercise T18: Specify the general real solution of each of the following linear homogeneous differential equations:

(a) $u^{(4)}(x) - 2u'''(x) + 5u''(x) - 8u'(x) + 4u(x) = 0$,

(b) $u'''(x) - 2u''(x) - 5u'(x) + 6u(x) = 0$,

(c) $u'''(x) - 3u''(x) + 4u(x) = 0$,

(d) $u^{(4)}(x) - u(x) = 0$,

(e) $u^{(4)}(x) + 4u(x) = 0$.

Tutorial date: Friday 21 May