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Karlsruhe, 1 June 2010

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Student Nr.: ...................

# Worksheet No.7
# Advanced Mathematics II

**Exercise 31:** Determine the general solution of the following differential equation

$$x^3 y'''(x) - 3x^2 y''(x) + 7xy'(x) - 8y(x) = 0\,.$$

**Exercise 32:** Find the general real solution of the following differential equation

$$2x^2 z''(x) + 4x^2 \left[z'(x)\right]^2 + 6xz'(x) + 5 = 0, \qquad x > 0.$$

Hint: Use the substitution $y(x) = e^{2z(x)}$.

**Exercise 33:** When solving oscillation problems in spherical coordinates (e.g. electromagnetic fields, acoustics, electron orbitals) one comes across Legendre's differential equation

$$(1 - x^2)f''(x) - 2xf'(x) + n(n+1)f(x) = 0, \quad n \in \mathbb{N}.$$

(a) Show for $n = 1$ that the function $f(x) = x$ is a solution of this differential equation.

(b) Determine another (linearly independent) solution for the case $n = 1$ using the the method of *reduction of the order*. You may assume $x > 1$.

**Exercise 34:** Determine the general complex solution of the differential equation system $u' = Au$ for the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 13 & -9 & -3 \end{pmatrix}$$

by reducing the system to a single ordinary differential equation. Calculate furthermore the complex eigenvalues of $A$. What do you notice?

**Exercise 35:** Consider a chemical reaction $A + B \to 2C$. We denote the molar concentrations of the three substances by $a$, $b$ and $c$ respectively. At the beginning of the reaction there are only the two substances $A$ and $B$ present (with concentrations $a_0$ and $b_0$). The reaction rate is proportional to the product of the concentrations $a$ and $b$. This leads to the following system of differential equations:

$$\begin{aligned} c(t) &= 2(a_0 - a(t)) \\ c(t) &= 2(b_0 - b(t)) \\ c'(t) &= k\,a(t)\,b(t) \qquad k \in \mathbb{R} \end{aligned}$$

(a) Is this a linear system of differential equations? Explain why or why not.

(b) Set up a single differential equation for $c(t)$ which does not contain $a(t)$ and $b(t)$.

(c) Determine $c(t)$ for $a_0 = \frac{1}{3}$, $b_0 = \frac{2}{3}$ and $k = 2$. You may assume $c(t) \leq \frac{2}{3}$.

**Due date:** Please hand in your homework until Thursday, 10 June, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

# Tutorial No.7
# Advanced Mathematics II

**Exercise T19:** Show that $y(x) = x$ fulfils the differential equation

$$(1 + x^2)y''(x) - 2xy'(x) + 2y(x) = 0\,,\ x \in \mathbb{R}$$

and determine another solution by means of the method of reduction of the order.

**Exercise T20:** Show that the function $y_1(x) = x^{-1}$ solves the differential equation

$$2x^2y''(x) + 3xy'(x) - y(x) = 0, \qquad x > 0.$$

Determine a second linearly independent solution by means of the method of reduction of the order.

**Exercise T21:** Determine the general real solutions of the differential equations for $x > 0$:

(a) $x^4u''''(x) + 6x^3u'''(x) - 2xu'(x) + 20u(x) = 0,$

(b) $x^3y'''(x) - 6x^2y''(x) + 15xy'(x) - 15y(x) = 0,$

(c) $y'''(x) + \frac{1}{x}y''(x) - \frac{2}{x^3}y(x) = 0,$

(d) $u'''(x) - \frac{2}{x}u''(x) + \frac{5}{x^2}u'(x) - \frac{5}{x^3}u(x) = 0.$

**Tutorial date:** Friday 4 June