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Karlsruhe, 8 June 2010

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Student Nr.: ....

Worksheet No.8
Advanced Mathematics II

Exercise 36: Solve the initial value problem

$$u'(t) = A \cdot u(t) \quad , \quad t \in [0, \infty) \quad , \quad u(0) = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \quad \text{with} \quad A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 3 \end{pmatrix}.$$

Exercise 37: Determine the general solution of the following differential equations for $x > 0$

(a) $y''(x) - y(x) = x$ using the *method of undetermined coefficients*,

(b) $y''(x) - y(x) = \frac{1}{x}$ using the *method of variation of parameters.*

Hint: The integral $\int \frac{e^x}{x} dx$ has no simple closed-form. You may leave this unevaluated in your result.

Exercise 38: Solve the initial value problem

$$y'''(x) + 3y''(x) + 4y'(x) - 8y(x) = (7 - 13x)e^x, \quad x \in \mathbb{R}$$

with $y(0) = y''(0) = 2$ and $y'(0) = 0$.

Exercise 39: Determine the general solution of the following differential equation

$$x^2y''(x) + xy'(x) - 4y(x) = 1 + x^2, \quad x > 0.$$

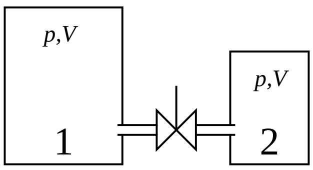
Exercise 40: *Two pressure tanks with different capacities V_1 and V_2 are linked by a pipe which is closed by a valve. Before opening the stop valve at time $t = 0$ the air in the tanks has two different pressures $p_1(0)$ and $p_2(0)$. By the ideal gas law $pV = nRT$ (n amount of substance, R gas constant, T temperature) assuming isothermal balancing we achieve the relation $\dot{p}V = \dot{n}RT$ and finally with flow resistance W of the pipe, $\dot{n} = Wp$ and the notation $a_{1,2} := \frac{RT}{WV_{1,2}}$ the following system for the model*

$$\begin{pmatrix} \dot{p}_1(t) \\ \dot{p}_2(t) \end{pmatrix} = \begin{pmatrix} -a_1 & a_1 \\ a_2 & -a_2 \end{pmatrix} \begin{pmatrix} p_1(t) \\ p_2(t) \end{pmatrix}, \quad t > 0.$$

Let $p_1(0) = 1$ bar, $p_2(0) = 9$ bar, $a_1 = 1$ bar/s and $a_2 = 3$ bar/s.

(a) In which tank and when is the pressure equal to two bar?

(b) Which pressure will be obtained when the system is completely balanced?



Due date: Please hand in your homework until Thursday, 17 June, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

Tutorial No.8
Advanced Mathematics II

Exercise T22:

(a) Determine a real fundamental system for

$$\begin{aligned} u' &= 2u + 2v \\ v' &= -\frac{1}{2}u + 2v\,. \end{aligned}$$

(b) Solve the following initial value problem

$$u'(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} u(x)\,,\ x \in \mathbb{R}\,, \quad u(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Exercise T23: Consider the inhomogeneous linear second-order ordinary differential equation

$$-15u(x) + 3xu'(x) + x^2u''(x) = 8x^{-3}, \qquad x > 0.$$

(a) Find a real-valued fundamental system of the associated homogeneous differential equation.

(b) Find a particular solution by the method of variation of parameters. Determine the general solution of the inhomogeneous problem.

Exercise T24: Determine the characteristic polynomial for the inhomogeneous third-order ordinary differential equation

$$y'''(x) + y''(x) + 4y'(x) + 4y(x) = f(x)\,,\ x \in \mathbb{R}\,.$$

For each of the following right hand sides determine an ansatz which reflects the structure of the right-hand side (method of undetermined coefficients):

$$\begin{array}{lll} f_1(x) = (4 + 12x)e^{2x} & f_2(x) = e^{-x} & f_3(x) = \sin(2x) \\ f_4(x) = x^2\cos(2x) & f_5(x) = xe^{-x}\sin(2x) & f_6(x) = (2 + x)\sin(2x) \end{array}$$

Use this to find a particular solution for f_1 and f_2.

Tutorial date: Friday 11 June