Tutorial No.2
Advanced Mathematics II

Exercise T4: Consider the vectors $u = (-2, 3, 1, 5)^\top, v = (1, 1, 2, 3)^\top, w = (-7, 3, -4, 1)^\top \in \mathbb{R}^4$.

(a) Construct the linear combinations $u + v + w,\ u - 3v + w,\ 3u - 2v\ ,\ 2u - 3v - w\ $.

(b) Show that every pair of the vectors is linearly independent.

(c) Are all three vectors linearly independent? Determine the dimension of $\text{span}\{u, v, w\}$.

Exercise T5: Verify that the three vectors

$$u = (1, 2, 0)^\top, \quad v = (0, 1, 0)^\top, \quad w = (1, 1, 1)^\top$$

constitute a basis of $\mathbb{R}^3$. Express the vectors

$$a = (1, 2, 3)^\top \quad \text{and} \quad b = (3, 2, 2)^\top$$

as a linear combination of u, v, w. Determine the coordinates of a and b with respect to the basis B.

Exercise T6:

(a) Check in each case whether the given vectors are linearly independent.

(i) $u = (1, 1, 0)^\top,\ v = (1, 0, 1)^\top,\ w = (0, 1, 1)^\top$;

(ii) $u = (1, 2, 3)^\top,\ v = (2, 3, 4)^\top,\ w = (3, 4, 5)^\top$;

(iii) $u^1 = (5, 0, 5, -4)^\top,\ u^2 = (0, 5, -5, -3)^\top,\ u^3 = (5, -5, 10, -1)^\top,\ u^4 = (-4, -3, -1, 5)^\top$.

(b) For which $\alpha \in \mathbb{R}$ are the vectors $(2, 1, 3)^\top$, $(1, -1, 2)^\top$ and $(-\alpha, 4, -3)^\top$ linearly dependent? For these values of α express the third vector as a linear combination of the first and the seond vector.

Tutorial date: Friday 23 April