Karlsruher Institut für Technologie
Institut für Algebra und Geometrie
PD Dr. F. Hettlich
Dr. S. Schmitt
Dipl.-Math. J. Kusch

51	52	53	54	55	$\sum$

Karlsruhe, 21.06.2011

Student Nr.: .....

Student Nr.: .....

Worksheet No.11
Advanced Mathematics II

Exercise 51: Consider the functions

$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases} \qquad \text{and} \qquad f(t) = \begin{cases} t-1, & 1 \leq t < 3, \\ 8-2t, & 3 \leq t < 4, \\ 0, & \text{else.} \end{cases}$$

(a) Rewrite the function f in a form without any case differentiation using the Heaviside function H.

(b) Sketch the graph of the function f and determine its Laplace transform.

Exercise 52: Let $f : [0, \infty) \to \mathbb{R}$ be the function with period 2 that is given in the interval $[0, 2)$ by

$$f(t) = \begin{cases} 1 & \text{for } 0 \leq t < 1, \\ 0 & \text{for } 1 \leq t < 2. \end{cases}$$

(a) Sketch the function's graph in the interval $[0, 10]$.

(b) Compute the Laplace transform of f.

Exercise 53: Solve the initial value problems by means of the Laplace transform:

(a) $y'''(t) - y''(t) + y'(t) - y(t) = e^{2t}, \qquad y(0) = 1,\ y'(0) = 0,\ y''(0) = -1$

(b) $x''(t) + 2x'(t) + x(t) = 3te^{-t}, \qquad x(0) = 4,\ x'(0) = 2,\ t \geq 0$

Exercise 54: Solve the initial value problem by means of the Laplace transform:

$$\begin{aligned} x(t) - 2y(t) + z(t) &= -2t, \\ -x'(t) + 3y'(t) - 2x(t) + y(t) &= 3 + t, \\ 3z''(t) - 5x'(t) - 2z(t) &= 0, \end{aligned}$$

$$x(0) = 1, \quad y(0) = -1, \quad z(0) = -3, \quad z'(0) = 2.$$

Exercise 55: Determine the solution $f(x)$ of Volterra's integral equation

$$f(x) - 2\int_0^x \cos(x-y) f(y) dy = e^x$$

by means of a Laplace transform. Hint: The convolution theorem might be useful here.

Due date: Please hand in your homework until Friday, July 1, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

Tutorial 11
Advanced Mathematics

Exercise T31: Express the following functions by means of Heaviside's function $H(t) = \begin{cases} 0, & t > 0 \\ 1, & t \geq 0 \end{cases}$:

(a) $f(t) = \begin{cases} 1, & t \geq t_0 \\ 0 & \text{otherwise} \end{cases}$ (b) $f(t) = \begin{cases} 1, & t \leq t_0 \\ 0, & \text{otherwise} \end{cases}$ (c) $f_{\alpha,\beta}(t) = \begin{cases} 1, & \alpha \leq t < \beta \\ 0, & \text{otherwise} \end{cases}$

Determine $\mathcal{L}\left(\frac{1}{\beta-\alpha} f_{\alpha,\beta}\right)$ and its behaviour for $\alpha = 0$ and $\beta \to \alpha$.

Exercise T32: Solve the following initial value problems by means of the Laplace transform:

(a) $\begin{aligned} x'(t) &= 2y(t) + 1, \\ y'(t) &= -2x(t) + 2t, \end{aligned} \qquad x(0) = 0, \quad y(0) = 1.$

(b) $\begin{aligned} u''(t) &= v(t) - u(t), \\ v''(t) &= u(t) - v(t), \end{aligned} \qquad u(0) = u'(0) = v(0) = 0, \quad v'(0) = 1.$

Exercise T33: Determine the convolution $f * g$ for the following pairs of functions

(a) $f(t) = \sin t; \qquad g(t) = \begin{cases} 0, & 0 \leq t < 3 \\ 2, & t \geq 3 \end{cases}$

(b) $f(t) = t^2; \qquad g(t) = 1 - h(t-1)$

(c) $f(t) = \sinh t; \qquad g(t) = \sin 2t.$

Compute $\mathcal{L}(f * g)$ and compare the result to $\mathcal{L}f \cdot \mathcal{L}g$.

Please note the following deadlines with respect to the **AM2 exam on 23 July 2011**:

- It is possible to register starting from 5 July 2011.
- The online registration **has to be carried out by 19 July 2011**.
- It is possible to cancel the registration online until 19 July 2011. For later cancellations, please see the staff at the institute.

For detailed information regarding this course please check the page
`http://www.math.kit.edu/iag1/lehre/am22011s/en`

Tutorial date: Tuesday, June 28.