Karlsruher Institut für Technologie
Institut für Algebra und Geometrie
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Karlsruhe, 30.06.2011

Student Nr.: ....

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Worksheet No.12
Advanced Mathematics II

Exercise 56: Determine the convolution $f * g$ for the functions

$$f(t) = t^2 \qquad \text{and} \qquad g(t) = 1 - H(t-1), \qquad \text{where } H(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{otherwise} \end{cases} \text{ denotes Heaviside's function.}$$

Also find $\mathcal{L}(f * g)$ and compare the result with $\mathcal{L}f \cdot \mathcal{L}g$.

Exercise 57: Find the preimage of

a) $\dfrac{3}{(s+3)(s+1)}$ b) $\left(\dfrac{2e^{-s}}{s} - \dfrac{3e^{-2s}}{s}\right)\dfrac{s}{s^2+9}$

by using partial fraction decomposition as well as the convolution theorem.

Exercise 58: Consider the function $f : \mathbb{R}^3 \to \mathbb{R}$ with

$$f(x_1, x_2, x_3) = x_1 \cos(x_2)\cos(x_3), \quad x_1, x_2, x_3 \in \mathbb{R}.$$

Determine the gradient $\nabla f(x)$ and the Hessian matrix $H_f(x) = \left(\dfrac{\partial^2 f}{\partial x_i \partial x_j}(x)\right)_{i,j=1,2,3}$

Exercise 59: Consider the scalar function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x) := x_1^2 x_2$, and a vector $d := (\cos\varphi, \sin\varphi)^\top$, $\varphi \in [0, 2\pi)$.

(a) Determine the gradient ∇f and the scalar product $d \cdot \nabla f$ at x.

(b) Calculate the directional derivative $\dfrac{\partial f}{\partial d}(x)$ using Definition 4.14.

Exercise 60: Find the derivative of the function $f : \mathbb{R}_{>0} \to \mathbb{R}$ given by

$$f(s) := \int_{1/s}^{s^2} \frac{\sin(st)}{t}\, dt\,,$$

applying partial derivatives of $g(x, y, z) = \int_x^y \frac{\sin(zt)}{t}\, dt$ and the chain rule.

Due date: Please hand in your homework until Friday, July 08, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

Tutorial 12
Advanced Mathematics

Exercise T34: Determine the convolution of the functions f and g with

a) $(\mathcal{L}f)(s) = \dfrac{1}{s^2(s^2+1)}$, b) $(\mathcal{L}g)(s) = \dfrac{1}{(s^2+1)^2}$.

Exercise T35: Let a function $f : \mathbb{R} \times (0, \infty) \to \mathbb{R}$ be given by $f(x, y) = y^x$.

a) Compute the gradient and the Hessian matrix of f.

b) Calculate the directional derivative of f at the point $\hat{x} = (2, 1)$ in the direction $a = \frac{1}{\sqrt{2}}(1, 1)^\top$.

Exercise T36:

(a) Determine all partial first derivatives of the function

$$F(x, y) = \int_1^{\sqrt{x}} \frac{1}{\tau} \cos(\tau^2 y \pi) d\tau, \quad x, y \geq 1 \,.$$

(b) Compute the value $g'(2)$ for the function $g(t) = \int_1^{\sqrt{t}} \frac{1}{\tau} \cos(\tau^2 t^2 \pi) d\tau, \quad t \geq 1 \,.$

Hint: Use $g(t) = F(t, t^2)$ and the chain rule.

For detailed information regarding this course please check the page
`http://www.math.kit.edu/iag1/lehre/am22011s/en`

Tutorial date: Tuesday, July 05