Karlsruher Institut für Technologie
Institut für Algebra und Geometrie
PD Dr. F. Hettlich
Dr. S. Schmitt
Dipl.-Math. J. Kusch

6	7	8	9	10	Σ

Karlsruhe, 12.04.2011

Student Nr.: ....

Student Nr.: ....

Worksheet No.2
Advanced Mathematics II

Exercise 6: Let

$$u = \begin{pmatrix} 1 \\ -2 \\ -1 \\ 0 \end{pmatrix}, \qquad v = \begin{pmatrix} 2 \\ 0 \\ 3 \\ -1 \end{pmatrix}, \qquad w = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \qquad z = \begin{pmatrix} 2 \\ 2 \\ 3 \\ -3 \end{pmatrix}$$

be vectors in $\mathbb{R}^4$.

(a) Compute the following linear combinations: $u + v - z$, $2v - (w - z)$, $2u - v + 2w$.

(b) Show that the vectors u, v, w are a basis of the subspace span $\{u, v, w\}$.

(c) What is the dimension of the subspace span $\{u, v, w, z\}$?

Exercise 7: For which $\alpha \in \mathbb{R}$ is $x = (-7, \alpha, 2)^\top$ a linear combination of $a^{(1)} = (1, 2, 4)^\top$, $a^{(2)} = (-2, 1, 2)^\top$ and $a^{(3)} = (3, 1, 2)^\top$? Determine all possible linear combinations of x.

Exercise 8: Let P_n denote the vector space of real polynomials of degree up to n.

(a) Show that the elements $e_k := (1 + x)^k$, $k = 0, 1, 2, 3$ form a basis of P_3. What is the dimension of this vector space?

(b) Express the polynomial $y = x^3 + 2x^2 + 1$ as a linear combination of the "basis vectors" e_k, $k = 0, 1, 2, 3$.

Hint: The "zero vector" in P_3 is the polynomial $0 \cdot x^3 + 0 \cdot x^2 + 0 \cdot x + 0$.

Exercise 9:

(a) Check in each case whether the given vectors are linearly independent.

(i) $u = (1, 1, 0)^\top$, $v = (1, 0, 1)^\top$, $w = (0, 1, 1)^\top$;

(ii) $u = (1, 2, 3)^\top$, $v = (2, 3, 4)^\top$, $w = (3, 4, 5)^\top$;

(iii) $u^1 = (5, 0, 5, -4)^\top$, $u^2 = (0, 5, -5, -3)^\top$, $u^3 = (5, -5, 10, -1)^\top$, $u^4 = (-4, -3, -1, 5)^\top$.

(b) For which $\alpha \in \mathbb{R}$ are the vectors $(2, 1, 3)^\top$, $(1, -1, 2)^\top$ and $(-\alpha, 4, -3)^\top$ linearly dependent? For these values of α express the third vector as a linear combination of the first and the seond vector.

Exercise 10: $C[0, 1]$ denotes the vector space of continuous functions on the closed interval $[0, 1]$. Let U be a subspace of $C[0, 1]$ spanned by two polynomials $b^{(1)}(x) = 1$ and $b^{(2)}(x) = x - \frac{1}{2}$. We define $y(x) := \sqrt{x} \in C[0, 1]$ and the scalar product of two functions by

$$\langle f, g \rangle := \int_0^1 f(x)\overline{g(x)}\, dx \in \mathbb{C}\,.$$

(a) Find a linear combination $c = a_1 b^{(1)} + a_2 b^{(2)} \in U$, such that $c(0) = y(0)$ and $c(1) = y(1)$.

(b) Determine $d \in U$ with smallest distance to y, i.e. the distance vector $e = d - y$ must be orthogonal to $b^{(1)}$ and $b^{(2)}$. Draw the graphs of y and the approximations c and d of it in $[0, 1]$ in a figure.
Remark: The Finite Element Method includes the computing of orthogonal approximations like d.

Due date: Please hand in your homework until Friday, April 29, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

Tutorial 1
Advanced Mathematics

Exercise T4: Let

$$u = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}, \qquad v = \begin{pmatrix} -9 \\ 2 \\ 4 \end{pmatrix}, \qquad w = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$$

be vectors in $\mathbb{R}^3$.

(a) Compute the following linear combinations of these vectors: $u + w$, $v - 3u$, $2u - v + w$.

(b) Show that every pair of vectors in the set $\{u, v, w\}$ are linearly independent.

(c) Are the three vectors also linearly independent as a triple?

Exercise T5: Check whether each of the following sets of vectors is linearly independent, and determine the dimension of $\text{span}\{u, v, w\}$.

(a) $u = (-1, 4)^\top,\ v = (2, 8)^\top,\ w = (3, 4)^\top$,

(b) $u = (-1, 4, 3)^\top,\ v = (1, 5, 2)^\top,\ w = (2, 1, 2)^\top$,

(c) $u = (-1, 4, 3)^\top,\ v = (1, 5, 2)^\top,\ w = (2, 1, -1)^\top$.

Exercise T6: For which values of the parameters $a, b \in \mathbb{R}$ are the vectors

$$u = \begin{pmatrix} a \\ 1 \\ 2 \end{pmatrix}, \quad v = \begin{pmatrix} 5 \\ 6 \\ b \end{pmatrix}, \quad w = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$$

linearly dependent?

For detailed information regarding this course please check the page
`http://www.math.kit.edu/iag1/lehre/am22011s/en`

Tutorial date: Tuesday, April 26.