Karlsruher Institut für Technologie
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Karlsruhe, 28.04.2011

Student Nr.: ....

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Worksheet No.3
Advanced Mathematics II

Exercise 6: Consider the plane $E : 4x_1 + x_3 + 8 = 0$, the point $P = (2|1|1)$ and the line $H : x(\lambda) = (4, 3, -2)^\top + \lambda(3, 1, -1)^\top$, $\lambda \in \mathbb{R}$.

(a) Determine a line G through P that is orthogonal to E.

(b) Determine the distance from P to E as well as the point Q in E closest to P.

(c) Determine the point at which the line H intersects E and the point R on H, that is closest to P.

Exercise 7: Let the points $P = (2|1|0)$, $Q = (1|3|-1)$ and $R = (0|2|0)$ be given.

(a) Represent the plane E through the points P, Q and R in both parametric and normal form.

(b) Does the line $G : x(u) = (-2, -7, 0)^\top + u(3, 2, 1)^\top$ intersect the plane E? If so determine the point and angle of intersection.

(c) Compute the orthogonal projection of the direction vector $(3, 2, 1)^\top$ of the line G onto the normal vector of the plane E. Using this and the intersection point of G and E determine the projection H of the line G onto E.

Exercise 8: Let the line G and the set E in $\mathbb{R}^4$ be given by $G : \; x = (0, 1, 0, -2)^\top + \lambda\,(2, 2, 1, 0)^\top \,, \; \lambda \in \mathbb{R}\,, \quad E : \; 3x_1 + 4x_4 = 4\,.$

(a) Determine the intersection of G and E.

(b) Find the set F of the type $a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = c$ that contains the point $(1|0|0|0)$ and is perpendicular to G.

(c) Determine all the points on G that have the same distance to the set E as to the set F from part (b).

Exercise 9: Consider the three matrices

$$A = \begin{pmatrix} 1 & 5 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 & 2 \\ 1 & 2 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 4 & -1 \\ 0 & 3 \\ 2 & 1 \end{pmatrix}.$$

Decide which of the following products are defined and compute them if possible:

$$AB, BA^\top, CA, CA^\top, C^\top A^\top, B^\top C^\top A^\top, (BA^\top)^\top C^\top, (CB)^\top A.$$

Exercise 10: For 3 times kitchen duty (K) and once swabbing the deck (S), poor seaman Hein B. obtains one Euro (E) and a pudding (P) from his captain. Moreover, Hein gets a pudding for four hand made fishing rods (R) and once swabbing the deck. For two fishing rods and four fish (F) he can afford a lemonade (L) in the harbour bar. Finally, for α fish and one kitchen duty, his captain tells one of his thrilling cock-and-bull stories (G). Here, $\alpha \in \mathbb{R}$ is a parameter depending on the captain's mood. Find the matrix A_α representing the linear mapping $(E, P, L, G) \mapsto (K, S, R, F)$. For which α is this mapping invertible? If it is invertible, compute the inverse and describe its meaning for kitchen duty, swabbing, fishing rods and fish.

Due date: Please hand in your homework until Friday, May 6, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

Tutorial 3
Advanced Mathematics

Exercise T4: Let

$$E : x_1 - x_3 = 0 \qquad \text{and} \qquad F : x_1 + 2x_2 + x_3 = 4.$$

be planes in $\mathbb{R}^3$

(a) Find the intersection line G between E and F.

(b) For another straight line H, which lies neither in E nor in F, exist only the following possibilitites:

- it intersects E in only one point and also F in only one point. (Find the angles of intersection.)
- it intersects one of them in only one point, but doesn't intersect the other at all,
- it doesn't intersect any of them.

Construct an example for each of these possibilities and visualize the geometric position of the planes and the straight line.

Exercise T5: Let the line $G : x(s) = (5, 1, -1)^\top + s(4, 0, -3)^\top,\ s \in \mathbb{R}$, and the two points $P = (2|0|2)$ and $Q = (0|2|2)$ be given.

(a) Determine a parametric representation of the line H that passes through P and Q.

(b) Determine the point R on G such that the plane E_1 passing through P, Q and R is parallel to the plane E_2 given by the equation $2x_1 + 2x_2 - 3x_3 = 0$. How far apart are the two planes?

(c) At what angle does G intersect the two planes?

Exercise T6: Which of the products $AB, AX, BX, X^\top A, (A^\top X)^\top B, B^\top X, XX^\top$ involving the matrices given below are well defined? Where appropriate determine the size of the resulting matrix and then evaluate the product.

$$A = \begin{pmatrix} 3 & 7 \\ 2 & 8 \\ 3 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} -2 & 8 \\ 6 & 5 \end{pmatrix} \qquad X = \begin{pmatrix} 8 \\ 4 \\ 1 \end{pmatrix}.$$

For detailed information regarding this course please check the page
`http://www.math.kit.edu/iag1/lehre/am22011s/en`

Tutorial date: Tuesday, May 3.