Karlsruher Institut für Technologie
Institut für Algebra und Geometrie
PD Dr. F. Hettlich
Dr. S. Schmitt
Dipl.-Math. J. Kusch

16	17	18	19	20	$\sum$

Karlsruhe, 05.05.2011

Student Nr.: ....

Worksheet No.4
Advanced Mathematics II

Exercise 16: Evaluate the matrix products AB^*, $BC^\top$ and $C^\top BA^\top$ for

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2i & 0 \\ 3 & 0 & 3 \\ 0 & 4i & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \\ 4 & 3 & 2 \\ 5 & 4 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} i & 0 & 0 \\ 2 & 2i & 0 \\ 3i & 3 & 3i \\ 0 & 4i & 4 \\ 0 & 0 & 5i \end{pmatrix}.$$

Exercise 17: Determine matrices corresponding to the following linear maps:

(a) $f : \mathbb{R}^3 \to \mathbb{R}^3, \quad f(x_1, x_2, x_3) = \begin{pmatrix} 3x_3 + x_1 + x_2 \\ 2x_2 + x_3 - x_1 \\ 2x_1 - 3x_2 - x_3 \end{pmatrix}$

(b) Let P_3 be the space of polynomials of degree up to 3, and let $g : P_3 \to P_3$ map each polynomial onto its derivative. The matrix representation should be with respect to the basis of monoms $\{1, x, x^2, x^3\}$.

Exercise 18: Specify those $\alpha \in \mathbb{R}$ for which the matrix

$$A = \begin{pmatrix} 1 & 0 & -2 & -2 \\ 2 & -2 & 1 & 2 \\ 0 & \alpha & 3 & -1 \\ -1 & 1 & 0 & 1 \end{pmatrix} \in \mathbb{R}^{4\times 4}$$

is regular and determine the appropriate inverse matrix in dependency on α!

Exercise 19: Determine the matrix $A \in \mathbb{R}^{2\times 2}$ with $Ax = y$ and $Ay = x$ for $x = (5, -5)^\top$ and $y = (-1, -7)^\top$. How does the inverse of A look like?

Exercise 20: Consider the parametric matrices

$$A = \frac{1}{9}\begin{pmatrix} 1 & -4 & 8 \\ -4 & 7 & \alpha \\ 8 & \alpha & 1 \end{pmatrix} \quad \text{and} \quad B = \frac{1}{7}\begin{pmatrix} -1 & -4+4i & 4 \\ -2\beta & 3 & \beta \\ 4 & 2-2i & 5 \end{pmatrix}.$$

(a) Determine the values of the parameters $\alpha \in \mathbb{R}$ and $\beta \in \mathbb{C}$ for which A is orthogonal and B is unitary.

(b) Using the values of α and β determined in a), compute: the lengths of the vectors Ax, Ay, Bx, Bz; the angle between Ax and Ay; and the scalar product $(Bx) \cdot (Bz)$, where $x = (4, 3, 0)^\top$, $y = (-3, 4, 0)^\top$ and $z = (2, i, -2)^\top$.

Due date: Please hand in your homework until Friday, May 13, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

Tutorial 4
Advanced Mathematics

Exercise T10: Given be two matrices representing a reflection and a rotation, respectively, namely

$$S = \frac{1}{3}\begin{pmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix} \quad \text{and} \quad R = \frac{1}{2}\begin{pmatrix} 1 & 0 & \sqrt{3} \\ 0 & 2 & 0 \\ -\sqrt{3} & 0 & 1 \end{pmatrix}.$$

(a) Show that S and R are orthogonal matrices.

Exercise T11: Given the matrices

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 1 \\ 2 & 3 & 4 \end{pmatrix},$$

calculate the inverses of A and B and find a matrix $D \in \mathbb{R}^{3\times 3}$, such that $ADB = C$.

Hint: The matrix multiplication is not *commutative*, i.e. $AB \neq BA$ in general.

Exercise T12:

(a) Find all fixed points of the matrix

$$A = \begin{pmatrix} -2 & 2 & 1 \\ 2 & -4 & 3 \\ 1 & 3 & -3 \end{pmatrix},$$

(i.e. all $x \in \mathbb{R}^3$ which satisfy $Ax = x$) using Gaussian elimination.

(b) The fixed points of A all lie on a straight line g. Find a vector w orthogonal to g. Calculate the angle between Aw and g.

(c) Let w be an arbitrary vector orthogonal to g. Show that then also Aw is orthogonal to g.

(d) Give a reason why $x \mapsto Ax$ cannot be a rotation around g.

For detailed information regarding this course please check the page
`http://www.math.kit.edu/iag1/lehre/am22011s/en`

Tutorial date: Friday, May 06.