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Karlsruhe, 12.05.2011

Student Nr.:

Worksheet No.5 Advanced Mathematics II

Exercise 21: Given two vectors $u, v \in \mathbb{C}^n$ with $1 + v^\top u \neq 0$. Show the matrix

$$B := I_n - \frac{uv^\top}{1 + v^\top u}$$

satisfies

$$(I_n + uv^\top)B = I_n = B(I_n + uv^\top).$$

Exercise 22: The linear map $\Phi : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is given by the matrix

$$A = \frac{1}{2} \begin{pmatrix} 7 & 10 \\ -5 & -7 \end{pmatrix}$$

with respect to the standard basis $E = \{e^{(1)}, e^{(2)}\}$. Another basis $B = \{b^{(1)}, b^{(2)}\}$ of the vector space \mathbb{R}^2 is given by $b^{(1)} = (-1, 1)^\top$ and $b^{(2)} = (-3, 2)^\top$.

- (a) Determine the basis transformation matrix from B to E , i.e. a matrix which transforms the vectors $x = \alpha_1 b^{(1)} + \alpha_2 b^{(2)} = (\alpha_1, \alpha_2)_B^\top$ with respect to the basis B into the representation $x = \beta_1 e^{(1)} + \beta_2 e^{(2)} = (\beta_1, \beta_2)_E^\top$.
- (b) Determine the basis transformation matrix from E to B , i.e. a matrix which transforms the vectors with respect to the standard basis E into the vectors with respect to the basis B .
- (c) Determine the matrix of the map Φ with respect to the basis B . Describe Φ geometrically.

Exercise 23: Evaluate the determinants

$$(a) D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}, \quad (b) D = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{vmatrix}, \quad (c) D = \begin{vmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{vmatrix}.$$

Exercise 24: Compute the determinant of the matrix $C := (AB)^{-1}$ with

$$A := \begin{pmatrix} 1 & 2 & 3 & -2 & 2 \\ 3 & -1 & 4 & 2 & 0 \\ 8 & -3 & -2 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } B := \begin{pmatrix} -1 & 3 & -2 & 2 & -1 \\ 0 & 2 & -3 & -3 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 2 & 5 \\ 0 & 2 & 0 & 0 & 6 \end{pmatrix}.$$

Exercise 25: Let

$$A = \begin{pmatrix} 1 & \alpha & -2 \\ 2 & -1 & \alpha - 1 \\ -1 & \alpha + 1 & 3 \end{pmatrix} \in \mathbb{R}^{3 \times 3}.$$

- a) For which $\alpha \in \mathbb{R}$ is A regular?
- b) For which $\alpha \in \mathbb{R}$ is the linear system $Ax = b$ with $b = (-2, 1, 3)^\top$ solvable? Determine the set of solutions.

Due date: Please hand in your homework until Friday, May 20, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1-3).

Tutorial 5 Advanced Mathematics

Exercise T13: Calculate the determinant of a real-valued 4×4 matrix

$$\begin{vmatrix} 1 & 2 & \pi & 1 \\ 2 & 4 & 1 & 0 \\ 1 & 0 & 0 & 3 \\ 2 & 4 & -1 & 0 \end{vmatrix}$$

- (a) using the expansion rule across the 3rd row,
- (b) using the expansion rule across the 4th column,
- (c) via Gaussian elimination.

Exercise T14: Given

$$A = \begin{pmatrix} 5 & i-1 & 7 & -4 \\ 5 & \frac{1}{2}(1-i) & 5 & -3 \\ 4 & 0 & 4 & -2 \\ 0 & 0 & 1 & -1 \end{pmatrix} \in \mathbb{C}^{4 \times 4} \quad \text{and} \quad B = \begin{pmatrix} 1 & -i-1 & -2 & -2 \\ 2 & -i-1 & -4 & -4 \\ 3 & -5i-5 & -4 & -4 \\ 4 & -7i-7 & -6 & -7 \end{pmatrix} \in \mathbb{C}^{4 \times 4}.$$

Determine $\det(A)$ and $\det(B)$, as well as $\det(AB^*)$ and $\det(A^{-1}B)$.

Exercise T15: Compute the determinant of the matrix

$$A = \begin{pmatrix} 3 & \alpha-1 & -\alpha+3 \\ 0 & \alpha+3 & -4 \\ 0 & 2 & \alpha-3 \end{pmatrix} \in \mathbb{R}^{3 \times 3}.$$

For which α is the linear system of equations $A_\alpha x = b$ for $b = (9, 4, -2)^\top$ solvable?

For detailed information regarding this course please check the page
<http://www.math.kit.edu/iag1/lehre/am22011s/en>