Karlsruher Institut für Technologie
Institut für Algebra und Geometrie
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26	27	28	29	30	$\sum$

Karlsruhe, 19.05.2011

Student Nr.: ....

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Worksheet No.6
Advanced Mathematics II

Exercise 26:

(a) Compute the characteristic polynomial $p(\lambda) = \det(A - \lambda I)$ of A for $\lambda \in \mathbb{R}$, where

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 3 \end{pmatrix}.$$

(b) Determine the eigenvalues, i.e. zeros $\lambda_1, \lambda_2, \lambda_3$ of the polynomial $p(\lambda)$.

(c) Find the eigenvectors, i.e. the solution set of the homogeneous system of linear equation $(A - \lambda_j I)x = 0$ for $j = 1, 2, 3$.

Exercise 27: Determine the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 3 & 0 & 2 \\ 0 & 1 & a \\ 0 & 2 & 2a \end{pmatrix}$ with $a \in \mathbb{R}$.

Exercise 28: Let the linear mapping $\Phi : \mathbb{R}^3 \to \mathbb{R}^3$ be a rotation around the x_2-axis by the angle $\frac{\pi}{3}$ and let the linear mapping $\Psi : \mathbb{R}^3 \to \mathbb{R}^3$ be a reflection on the plane $E :\ x_1 + 2x_2 = 0$.

(a) Determine the matrices A of Φ and B of Ψ with respect to the standard basis. Check if A and B are orthogonal matrices.

(b) Compute the eigenvalues of A and B, as well as the eigenvectors to the real eigenvalues.

Exercise 29:

(a) Determine the eigenvalues of $A = \begin{pmatrix} 5 & 0 & 4 \\ 0 & -6 & 0 \\ 1 & 0 & 2 \end{pmatrix}$.

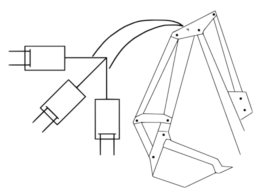
(b) Let $p(x) = c_3x^3 + c_2x^2 + c_1x + c_0$ be the characteristic polynomial of A. Show that $p(A) = 0$, i.e. $c_3A^3 + c_2A^2 + c_1A + c_0I_3 = 0$. (This is true in general for every square matrix and its characteristic polynomial!)

(c) From $p(A) = 0$, determine the inverse matrix A^{-1}.

Exercise 30: By a measurement rosette (Dehnmessstreifen-Rosette) on a surface the extension state of a hydraulic shovel may be determined in the form of a strain tensor with respect to the x_1, x_2, x_3 coordinate system. We are interested in the principle strains (Hauptdehnungen) as they determine the maximal forces in the corresponding principle strain axes in isotropic materials.

Determine for the given tensor

$$\varepsilon = \frac{1}{50} \begin{pmatrix} 142 & -144 & 0 \\ -144 & 58 & 0 \\ 0 & 0 & -125 \end{pmatrix}$$



the principle strains (i.e. the eigenvalues of ε) and compute for each principle strain a normed eigenvector as corresponding principle strain axis. Determine the angles between the principle strain axes, discuss whether they form a basis of $\mathbb{R}^3$ and determine their angle to the standard basis.

Due date: Please hand in your homework until Friday, May 27, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

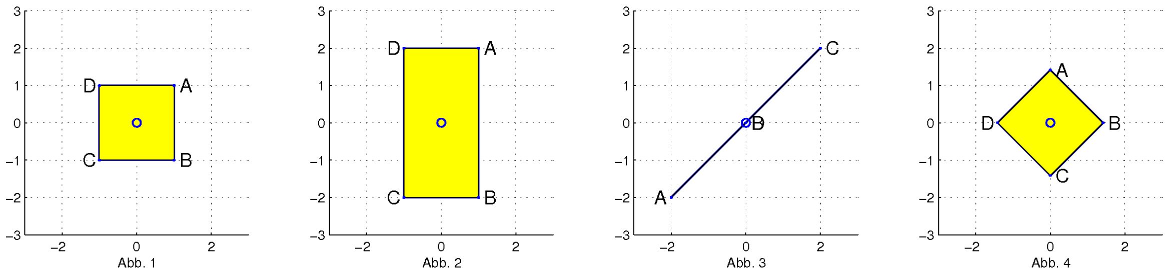
Tutorial 6
Advanced Mathematics

Exercise T16: Determine all eigenvalues of the square matrices

$$\text{(a)} \quad A = \begin{pmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{pmatrix}, \qquad \text{(b)} \quad B = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 3 \end{pmatrix},$$

and an eigenvector to every real eigenvalue λ.

Exercise T17: Figure 1 shows a square around the origin with the vertices A, B, C, D. The following figures show the images of the square obtained by applying different linear mappings $\Phi_{1,2,3} : \mathbb{R}^2 \to \mathbb{R}^2$:



Determine the eigenvalues of the linear mappings. Sketch the eigenvectors if possible.

Exercise T18: Let

$$C = \begin{pmatrix} 5 & 4 & -5 & 5 \\ -1 & 1 & 2 & -2 \\ -3 & -4 & 7 & -5 \\ -4 & -5 & 7 & -5 \end{pmatrix}.$$

Calculate all eigenvalues and eigenvectors of C.

For detailed information regarding this course please check the page
`http://www.math.kit.edu/iag1/lehre/am22011s/en`

Tutorial date: Tuesday, May 24.