Karlsruher Institut für Technologie
Institut für Algebra und Geometrie
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31	32	33	34	35	$\sum$

Karlsruhe, 26.05.2011

Student Nr.: ....

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Worksheet No.7
Advanced Mathematics II

Exercise 31: Let the following differential equation for $x > 0$ be given:

$$y'''(x) - \frac{2}{x}y''(x) + \frac{5}{x^2}y'(x) - \frac{5}{x^3}y(x) = 0$$

Check, if the following functions are solutions of this differential equation:

(a) $y_1(x) = \sin(x^2)$,

(b) $y_2(x) = x$,

(c) $y_3(x) = \exp(\frac{2}{x})$,

(d) $y_4(x) = x^2 \cos(\ln(x))$.

Exercise 32: Find the real-valued general solution for each of the following homogeneous ordinary differential equations:

(a) $x^2u''(x) - 5xu'(x) + 13u(x) = 0$, $x > 0$,

(b) $u''(x) - 5u'(x) + 13u(x) = 0$, $x > 0$,

(c) $u'''(x) - \frac{3}{x}u''(x) + \frac{7}{x^2}u'(x) - \frac{8}{x^3}u(x) = 0$, $x > 0$.

Exercise 33: Solve the initial value problem

$$x^4u''''(x) + 6x^3u'''(x) - 2xu'(x) + 20u(x) = 0 \quad x > 0,$$

$u(1) = 0$, $u'(1) = 4$, $u''(1) = 5$, $u'''(1) = 24$.

Exercise 34: Determine the general real-valued solution of the homogeneous differential equation

$$y'''(x) + 2y''(x) + 2y'(x) + y(x) = 0$$

using the exponential ansatz $y(x) = e^{\lambda x}$, $\lambda \in \mathbb{C}$.

Exercise 35: Show that $u(x) = e^{x^2}$ solves the homogeneous differential equation

$$u''(x) - 2xu'(x) - 2u(x) = 0, \quad x \in (0, \infty) .$$

Determine a second non-trivial solution by means of the method of reduction of the order.

Note: There is no explicit form for the antiderivative of $\int e^{-x^2} dx$. So just keep the integral in the answer.

Due date: Please hand in your homework until Friday, June 03, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

Tutorial 7
Advanced Mathematics

Exercise T19: Determine the general solution of the linear homogeneous differential equations with constant coefficients:

(a) $y'''(x) - 3y''(x) - y'(x) + 3y(x) = 0\,,\ x \in \mathbb{R}\,,$

(b) $y'''(x) + 7y''(x) + 19y'(x) + 13y(x) = 0\,,\ x \in \mathbb{R}\,.$

Exercise T20: Consider the homogeneous Euler differential equation

$$2x^3u'''(x) + Bx^2u''(x) + xu'(x) - 10u(x) = 0, \quad x > 0.$$

(a) Determine $B \in \mathbb{R}$ so that $u_1(x) = x^{\frac{5}{2}}$ is a solution of the differential equation.

(b) Determine the general solution of the differential equation for the computed constant B from part (a).

Exercise T21: Show that $y(x) = x$ fulfils the differential equation

$$(1 + x^2)y''(x) - 2xy'(x) + 2y(x) = 0\,,\ x \in \mathbb{R}$$

and determine other solution by means of the method of reduction of the order.

For detailed information regarding this course please check the page
`http://www.math.kit.edu/iag1/lehre/am22011s/en`

Tutorial date: Tuesday, May 31.