Karlsruher Institut für Technologie
Institut für Algebra und Geometrie
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Karlsruhe, 09.06.2011

Student Nr.: ....

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Worksheet No.9
Advanced Mathematics II

Exercise 41: Given be the inhomogeneous linear second-order ordinary differential equation

$$x^2y''(x) - 2xy'(x) + 2y(x) = x^3 \ln x, \quad x > 0.$$

The associated homogeneous differential equation has a solution of the form $y(x) = Ax + B$.

a) Determine the general solution of the homogeneous problem using the method of reduction of the order.

b) Determine a particular solution of the inhomogeneous problem by means of variation of the constants.

c) Solve the initial value problem for the inhomogeneous ordinary differential equation with $y(1) = y'(1) = 1$.

Exercise 42: Consider the inhomogeneous linear second-order ordinary differential equation

$$-15u(x) + 3xu'(x) + x^2u''(x) = 8x^{-3}, \qquad x > 0.$$

(a) Find a real-valued fundamental system of the associated homogeneous differential equation.

(b) Find a particular solution by the method of variation of parameters. Determine the general solution of the inhomogeneous problem.

Exercise 43: Determine by power series method the solution of following initial value problem for the Tshebysheff differential equation with index $n \in \mathbb{N}_0$:

$$(1 - x^2)u''(x) - xu'(x) + n^2u(x) = 0, \quad x \in (-1, 1), \quad u(0) = 1, \quad u'(0) = 0.$$

Show that

(a) all coefficients with odd power vanish,

(b) in case of even n the series truncates after the summand including x^n and

(c) in case of odd n the solution's radius of convergence is 1.

Exercise 44: Find the solution of the initial value problem

$$y'' - 2xy' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

by power series method and determine the radius of convergence.

Exercise 45: Use the generalized power series method, i.e., employ the ansatz $y(x) = \sum_{k=0}^{\infty} a_k x^{k+\lambda}$, $\lambda \notin \mathbb{N}_0$ to determine the general solution of the homogeneous differential equation

$$x^2y''(x) + x^2y'(x) - 2y(x) = 0.$$

Due date: Please hand in your homework until Friday, June 17, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1–3).

Tutorial 9
Advanced Mathematics

Exercise T25: For the inhomogeneous linear second-order ordinary differential equation

$$x^2y''(x) - \frac{3}{2}xy'(x) + y(x) = x^3 \ ,$$

determine

(a) the general solution of the homogeneous differential equation by means of reduction of the order. Use the fact that $y_1(x) = x^2$ solves the homogeneous problem.

(b) a particular solution and the general solution of the inhomogeneous differential equation by means of variation of the constants.

(c) a solution of the initial value problem with $y(1) = \frac{17}{5}$ and $y'(1) = \frac{21}{5}$.

Exercise T26: Determine a solution of the initial value problems applying the power series method. Find an explicit formula for the coefficients.

(a) $(2x - x^2)y''(x) + (1 - x)y'(x) = 0, \quad y(1) = 1, \quad y'(1) = 0$

(b) $(x^2 + 1)u''(x) - 6u(x) = 0, \quad u(0) = 0, \quad u'(0) = 1$

(c) $(x^2 + 1)u''(x) - 6u(x) = 0, \quad u(0) = 1, \quad u'(0) = 0$

Also determine the radius of convergence of the solution in part (c), as well as the general solution of the differential equation $(x^2 + 1)u''(x) - 6u(x) = 0$.

Exercise T27: Use the generalized power series method, i.e., employ the ansatz $y(x) = \sum_{k=0}^{\infty} a_k x^{k+\lambda}$ to determine the general solution of the differential equation

$$x^2y''(x) + x^2y'(x) - 2y(x) = 0.$$

Determine the coefficients explicitly.

For detailed information regarding this course please check the page
`http://www.math.kit.edu/iag1/lehre/am22011s/en`

Tutorial date: Tuesday, May 14