Karlsruher Institut für Technologie (KIT)
Institut für Algebra und Geometrie
PD Dr. F. Hettlich
Dipl.-Math. M. Mitschele

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Group

Karlsruhe, October 22, 2010

Name: ....

Matrikel-Nr.: ....

Exercise sheet 1
Advanced Mathematics III
for Mechanical Engineering

Question 1: Let a function $f:\mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = \dfrac{x^2 y}{x^4+y^2}$ for $(x,y) \neq (0,0)$, and $f(0,0) := 0$.

a) Give reasons why f can be continuously differentiated arbitrarily many times in $\mathbb{R}^2 \setminus \{(0,0)^\top\}$.

b) Show that f is discontinuous at the origin.

c) Prove that for f all directional derivatives exist at the origin.

Hint for c): Use $(c,s)^\top \in \mathbb{R}^2$ with $c^2+s^2=1$ as a direction.

Question 2: Consider functions $f:(0,\infty)\times(-\pi,\pi) \to \mathbb{R}^2$ with $f(r,\varphi) = (r\cos\varphi, r\sin\varphi)^\top$ and $g:\mathbb{R}^2 \setminus \{(0,0)^\top\} \to \mathbb{R}$ with $g(x,y) = \dfrac{xy}{x^2+y^2}$. Determine all partial derivatives of the function $h := g \circ f$

a) by means of the chain rule,

b) directly.

Question 3: Let a function $f:\mathbb{R}\times(0,\infty) \to \mathbb{R}$ be given by $f(x,y) = y^x$.

a) Compute the gradient and the Hessian matrix of f.

b) Determine the linearisation of f in the point $(1,1)^\top$, as well as the **Taylor polynomial of degree 2** of f in the point $(1,1)^\top$, which is given by

$$T_2^f\left((x,y);(1,1)\right) = f(1,1) + f'(1,1)\begin{pmatrix} x-1 \\ y-1 \end{pmatrix} + \frac{1}{2!}\begin{pmatrix} x-1 \\ y-1 \end{pmatrix}^\top H_f(1,1)\begin{pmatrix} x-1 \\ y-1 \end{pmatrix}.$$

c) Use the Taylor polynomial of part b) to compute approximately the value of the number $\alpha = 0.99^{1.01}$, i.e., determine $T_2^f\left((1.01, 0.99);(1,1)\right)$.

Question 4: Check whether the following differential equations of first order are exact and determine their solutions:

a) $(y+x) - (y-x)y' = 0$,

b) $(2xe^y - 1)\,dx + (x^2 e^y + 1)\,dy = 0$.

Question 5: Determine the solution of the following initial value problem in implicit form:

$$\cos y + 2xy + (x^2 - y - x\sin y)y' = 0, \qquad y(0) = \sqrt{2}.$$

Hint: Check whether the differential equation of first order is exact!

Deadline: Thursday, October 28, 2010 at 15:45 h

Information regarding
Advanced Mathematics III
for Mechanical Engineering

- Concerning the purchase of the script on AM III, please refer to Hr. PD Dr. Frank Hettlich.

- During this winter semester AM III will be accompanied by a "Saaltutorium", which is scheduled for Mondays, 17:30 h–19:00 h in the Gerthsen–Hörsaal. In the "Saaltutorium" the problems of the current exercise sheet shall be solved in groups. Students of higher semesters as well as staff from the Institut für Algebra und Geometrie will be available for answering work-related questions.

- You shall be awarded the "Übungstestat" for AM III provided that you

  - score a minimum total of 150 marks from exercise sheets 1 to 12, and

  - score a minimum of 5 marks each from at least 10 of these exercise sheets.

- Important dates:

  - On Friday, October 22, 2010, 11:45 h to 13:30 h, you can take a look at your written exam in AM I, AM II and/or AM III in the Hörsaal 37 (building 20.40). Please remember to bring along your student–ID.
  - The written exam in AM III will take place on Saturday, March 12, 2011.

- Our current office hours are:

PD Dr. Frank Hettlich	Dipl.–Math. Marc Mitschele
Mondays, 15:30 h–16:30 h	Tuesdays, 15:00 h–16:30 h
Room 4C-21	Room 4C-01
Allianz–Gebäude (05.20)	Allianz–Gebäude (05.20)

  Possible changes of our office hours will be communicated via our institute's website.

- This information and any further updates thereof can be obtained from our institute's website at:

  `www.math.kit.edu/iag1/lehre/hm3mach2010w/`