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Group

Karlsruhe, October 29, 2010

Name: ....

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Exercise sheet 2
Advanced Mathematics III
for Mechanical Engineering

Question 1: Determine the solution $y = y(x)$ of the initial value problem

$$\tan(xy) + xy + x^2 y' = 0 \ , \quad y(1) = \frac{\pi}{6} \ ,$$

in explicit form using an integrating factor that only depends on xy.

Hint: Note that the relations $\tan'(t) = 1 + \tan^2(t)$ and $\int \tan(t)\,dt = -\ln(\cos(t))$ for $|t| < \frac{\pi}{2}$ hold.

Question 2: Determine in implicit form the general solution of the non-linear first-order ordinary differential equation

$$2\frac{(u(x))^2}{x} + (1 + 2u(x)\,\ln x)\,u'(x) = 0 \ ,$$

for $x \in (0, \infty)$, by employing an integrating factor (Eulerian multiplier) $l : \mathbb{R} \to \mathbb{R}$ of the form $l(t)$ with $t = x^u$. Which solution curve runs through the point $p = (2, 1)$?

Question 3: Consider a twice continuously differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$. For a point $(a, b)^\top \in \mathbb{R}^2$, let the relations $f(a, b) = 0$ and $f_y(a, b) \neq 0$ hold. As is well known, one can solve the equation $f(x, y) = 0$ for y in a neighbourhood of $(a, b)^\top$, i.e. $y = g(x)$, with a continuously differentiable function g.

a) Show that g is twice differentiable and determine the second derivative of g.

b) Determine the second derivative $g''(0)$ for the case $f(x, y) = y + x \sin y = 0$, $(a, b) := (0, 0)$.

Question 4: Consider the function

$$f : \mathbb{R}^2 \to \mathbb{R} \ , \quad f(x_1, x_2) = e^{x_1 x_2} + x_1^2 + 2x_2^2 \ .$$

Show that the algebraic equation $f(x_1, x_2) = e^{\sqrt{6}} + 7$ can be solved for x_2 in a neighbourhood of the point $(\sqrt{3}, \sqrt{2})^\top$, i.e., that there exists a function ϕ such that $f(x_1, \phi(x_1)) = e^{\sqrt{6}} + 7$ for x_1 in a neighbourhood of $x_1 = \sqrt{3}$. Determine the Taylor polynomial of first order of ϕ near $x_1 = \sqrt{3}$.

Question 5: The algebraic equation $(x^2+y^2)^2 - y(3x^2-y^2) = 0$ determines a curve in implicit form. In certain points this equation can be locally solved uniquely for y or x, respectively, i.e., locally there holds $y = g(x)$ or $x = h(y)$. Determine

a) (as far as existent) the symmetries of this curve wrt. the x- and y-axes,

b) those points on the curve for which a unique solution $y = g(x)$ exists and for which in addition $y'(x) = 0$ holds,

c) the singular points of the curve, i.e., those points for which the equation cannot be solved uniquely for either x or y,

d) the points of intersection of this curve with the line $y = x$, as well as $y'(x)$ in these points, if they allow for a unique solution $y = g(x)$.

Deadline: Tuesday, November 9, 2010 at 11:30 h