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Group

Karlsruhe, November 5, 2010

Name: ....

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Exercise sheet 3
Advanced Mathematics III
for Mechanical Engineering

Question 1: Determine all stationary points, as well as their nature, of the following scalar-valued functions:

a) $f : \mathbb{R}^2 \to \mathbb{R},\ f(x, y) = xy + x - 2y - 2$,

b) $f : \mathbb{R}^2 \to \mathbb{R},\ f(x, y) = 4e^{x^2+y^2} - x^2 - y^2$,

c) $f : \mathbb{R}^3 \to \mathbb{R},\ f(x, y, z) = -\ln(x^2 + y^2 + z^2 + 1)$.

Question 2: Consider a wooden plate with a certain mass per area A. The task at hand is to build a box (without a lid) which is of maximal volume for a given fixed total mass. Determine the lengths x, y, z of the edges of the optimal box in dependence of the area A. Which maximal volume in litres can be attained for a box made from oak wood with 7.3 kg/m^2 when its total mass is fixed to 10 kg?

Question 3: In the set

$$M = \left\{(x, y) \in \mathbb{R}^2 : xy^2 - x^2 + 2x - y^2 - 1 = 0\right\} ,$$

there is exactly one point that has minimal distance from the origin. Determine the coordinates of this point.

Question 4: Determine the local extremal values of the scalar-valued function $f(x, y) = x^2 - \frac{xy}{2} + \frac{y^2}{4} - x$ on the set

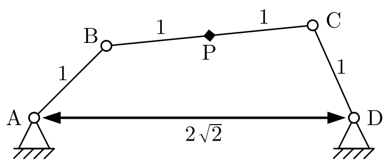
$$E = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 :\ x^2 + \frac{y^2}{4} \leq 1 \right\} .$$

For the investigation of the properties of the function f on the boundary of E, use the rule of Lagrange multipliers.

Question 5: Consider a rectangle of hinges $ABCD$: the two points $A(-a, 0)$ and $D(a, 0)$ shall be fixed, and there shall hold the relations $d(A, B) = d(C, D) = r$ and $d(B, C) = 2\ell$. We are interested in the locus of the midpoint $P(x, y)$ of the line BC. One can show that its coordinates satisfy the algebraic equation

$$(x^2 + y^2)(x^2 + y^2 + \ell^2 - r^2 - a^2)^2 + 4a^2y^2(x^2 + y^2 - r^2) = 0 .$$

For $r = \ell = 1$ and $a = \sqrt{2}$, determine the locus of the point P with a maximum value for its y-coordinate.



Hint: For solving the Lagrange equations, it is useful to set $u := x^2 + y^2$.

Remark: This so–called Watt linkage was invented by James Watt in 1784. It is still used today in order to keep a point P (in a neighbourhood of the origin), with a minimum amount of friction, on a straight line.

Deadline: Tuesday, November 16, 2010 at 11:30 h