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Group

Karlsruhe, November 12, 2010

Name:

Matrikel-Nr.:

Exercise sheet 4 Advanced Mathematics III for Mechanical Engineering

Question 1: Evaluate the following domain integrals:

- a) $\int_D (4x - y) d(x, y), \quad D = [0, 1] \times [-1, 2],$
- b) $\int_D \cos(x + y) d(x, y), \quad D = [0, \frac{\pi}{2}] \times [0, \pi],$
- c) $\int_D \frac{x^2 + e^y}{z + 1} d(x, y, z), \quad D = [-1, 2] \times [0, 1] \times [0, e - 1],$
- d) $\int_D (\ln x + y^2 e^z) d(x, y, z), \quad D = [1, 2]^3.$

Question 2: Sketch the surface of intersection D between the disc $x^2 + y^2 \leq 1$ and the half planes $y \geq -x$ and $x \leq 0$, and determine the area and the centre of gravity of D .

Question 3: Evaluate the domain integrals

$$(a) \quad \iint_B (x_1^2 - x_2^2) d(x_1, x_2), \quad \text{and} \quad (b) \quad \iint_S \frac{\sin(x_2)}{x_2} d(x_1, x_2);$$

the first on $B \subseteq \mathbb{R}^2$ which is bounded by the graphs of the functions $x_2 = x_1^2$ and $x_2 = x_1^3$, the second on $S \subseteq \mathbb{R}^2$ which is defined by $S := \{(x_1, x_2)^\top \in \mathbb{R}^2 : 0 \leq x_1 \leq x_2 \leq \frac{\pi}{2}\}$. Which variant proves to be more effective: integrating first w.r.t. x_1 or integrating first w.r.t. x_2 ?

Question 4: Let a domain $\Omega \subset \mathbb{R}^2$ be bounded by the four functions

$$\begin{aligned} y &= \frac{1}{x}, & x &\in [1/e, 1], & y &= \frac{e^2}{x}, & x &\in [1, e], \\ y &= x, & x &\in [1, e], & y &= e^2 x, & x &\in [1/e, 1]. \end{aligned}$$

Evaluate the integral

$$\iint_{\Omega} \ln(x) \ln(y) d(x, y)$$

by employing the variable transformation $x = \exp(u - v)$, $y = \exp(u + v)$, $(u, v) \in \mathbb{R}^2$, and the associated transformation formula.

Question 5: Find the volume of the body given by

$$\left\{ (x, y, z) \in \mathbb{R}^3 : \frac{x^2}{4} + \frac{y^2}{9} \leq 1, z \geq 0 \quad \text{and} \quad z \leq 2x + y + 10 \right\}.$$