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Group

Karlsruhe, November 19, 2010

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Exercise sheet 5
Advanced Mathematics III
for Mechanical Engineering

Question 1: Consider the rotational ellipsoid $E = \left\{(x,y,z)^\top \in \mathbb{R}^3 : \; x^2 + 4y^2 + z^2 \leq 9\right\}$. Evaluate the domain integral $\int\limits_E \left(x^2 + y + z^2\right) d(x,y,z)$ by means of

a) cylindrically polar coordinates,

b) spherical polar coordinates adapted to the ellipsoid.

Question 2: Determine the volumes of the following two bodies:

a) $K = \left\{(x,y,z)^\top \in \mathbb{R}^3 : \; x^2 + y^2 + z^2 \leq 1, \; x^2 + y^2 + (z-1)^2 \leq 1\right\}$,

b) $K = \left\{(x,y,z)^\top \in \mathbb{R}^3 : \; x^2 + y^2 \leq 1, \; x^2 + z^2 \leq 1\right\}$.

Question 3:

a) Let a function $f : [a,b] \to \mathbb{R}$ be continuously differentiable, and let $f \geq 0$ on $[a,b]$. Moreover, let a curve C be given by the parametrisation $x(t) = (f(t)\cos(t), f(t)\sin(t))^\top$, $t \in [a,b]$. Show that for the length L of this curve the relation $L = \int_a^b \sqrt{(f(t))^2 + (f'(t))^2}\, dt$ holds.

b) Let $a > 0$, and let a curve $x : [-\pi, \pi] \to \mathbb{R}^2$ be given by

$$x(t) = \begin{pmatrix} a(1+\cos(t))\cos(t) \\ a(1+\cos(t))\sin(t) \end{pmatrix}.$$

Sketch the curve and determine its unit tangent vector $\tau(t) := \dot{x}(t)/\|\dot{x}(t)\|_2$. Show that the curve is smooth. Evaluate the length of this curve.

Hint for b): For the proof of smoothnes of the curve, use the identities: $1 + \cos(t) = 2\cos^2\left(\frac{t}{2}\right)$, $\cos^2(t) - \sin^2(t) = \cos(2t)$, $\sin(2t) = 2\sin(t)\cos(t)$ and $\cos(s)\cos(t) = \frac{1}{2}(\cos(s-t) + \cos(s+t))$.

Question 4: Let a curve $C \subseteq \mathbb{R}^2$ be defined by $C = \left\{x(t) = \frac{1}{\sqrt{2}} e^t (\cos(t), \sin(t))^\top : \; t \in [0, 2\pi]\right\}$.

a) Evaluate the length L of curve C.

b) Determine for C a parametrisation in terms of its arclength $y : [0, L] \to C$.

c) Evaluate the line integral $\int\limits_C \frac{x_1 x_2}{x_1^2 + x_2^2}\, ds$.

Question 5: Evaluate the following line integrals:

a) $\int\limits_C f(x)\, ds$, wherein the function f is given by $f(x) = f(x_1, x_2) = \frac{1}{1+x_1+x_2}$, and the curve C is given by the parametrisation $x(t) = (t, \; \frac{1}{2}t^2, \; \frac{2\sqrt{2}}{3}t^{\frac{3}{2}})^\top, \;\; t \in [1,2]$,

b) $\int\limits_C f(x) \cdot ds$, wherein the function f is given by $f(x) = f(x_1, x_2, x_3) = (4x_1x_2x_3, \; x_3, \; x_1^2 x_2^2)^\top$, and the curve C is given by the parametrisation $x(t) = (t, t^2, 1)^\top, \;\; t \in [0,1]$,

c) $\int\limits_C f(x) \cdot ds$, wherein the function f is given by $f(x) = f(x_1, x_2, x_3) = (4x_1x_2x_3, \; x_3, \; x_1^2 x_2^2)^\top$, and the curve C is given by the parametrisation $x(t) = (t, t, 1)^\top, \;\; t \in [0,1]$.

Deadline: Thursday, November 25, 2010 at 15:45 h