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Group

Karlsruhe, December 17, 2010

Name: ....

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Exercise sheet 9
Advanced Mathematics III
for Mechanical Engineering

Question 1: Employing a separation of variables ansatz $u(x,y) = v(x)w(y)$, determine non-trivial solutions of the following partial differential equations:

a) $16u_{yyyy} + u_x = 0$,

b) $2yu_{xx} - (1 + y^2)u_y + 4yu = 0$.

Question 2: The telegrapher's equation $u_{tt} - u_{xx} + 2u_t + u = 0$ describes the temporal development of a signal voltage u at position $x > 0$ in a long transmission cable.
To be determined is the signal voltage $u(x,t)$, when at the boundary at $x = 0$ of the transmission cable a periodic signal of the form $u(0,t) = 3\sin(2t)$, for $t \geq 0$, is fed in. The signal voltage u is assumed to remain bounded for $x \to \infty$.
a) Show that a separation of variables ansatz of the form $u(x,t) = v(x) \cdot w(t)$ cannot yield a solution of this differential equation.
b) Solve the problem by means of the ansatz $u(x,t) = u_0 e^{-ax} \sin(2t - bx)$, with $a, b \in \mathbb{R}$ and $a > 0$.

Question 3: For the annulus $1 \leq x^2 + y^2 \leq 4$, solve the Dirichlet problem (in plane polar coordinates):

$$\begin{aligned} &r^2 u_{rr} + r u_r + u_{\varphi\varphi} = 0, \quad 1 < r < 2, \\ &u(1,\varphi) = 1 + 3\cos\varphi + 4\sin(2\varphi), \\ &u(2,\varphi) = 1 + 2\ln 2 + 6\cos\varphi + \sin(2\varphi). \end{aligned}$$

Also express the solution in terms of Cartesian coordinates and draw a sketch of the former.
Hint: Consider a separation of variables ansatz resp. consult the lecture notes provided.

Question 4: Employ the method of separation of variables to find the solution of the following problem of heat conduction:

$$u_t(x,t) = u_{xx}(x,t) \qquad \text{for} \quad 0 < x < 1 \ , \quad 0 < t \ ,$$

subject to the boundary conditions $u_x(0,t) = u_x(1,t) = 0$ and the initial condition $u(x,0) = \cos(7\pi x)$.

Question 5: Solve the following initial–boundary value problems:

$$\begin{aligned} u_{tt} &= 16u_{xx} \quad \text{for} \quad 0 < x < \pi \ , \quad 0 < t \ , \qquad u(0,t) = u(\pi,t) = 0 \quad \text{for} \quad t \geq 0 \ , \\ u(x,0) &= \frac{\pi x\,(\pi - x)}{8} \quad \text{for} \quad 0 \leq x \leq \pi \ , \qquad u_t(x,0) = 0 \quad \text{for} \quad 0 \leq x \leq \pi \ . \end{aligned}$$

Hints: Cf. problems discussed in the lectures. Use: $\dfrac{\pi x\,(\pi - |x|)}{8} = \displaystyle\sum_{k=1}^{\infty} \frac{\sin\left((2k-1)x\right)}{(2k-1)^3}$ for $x \in [0,\pi]$.

We wish you all a Merry Christmas
and a Happy New Year.

Deadline: Thursday, January 13, 2011 at 15:45 h