Exercise 3:

We identify $\mathbb{R}^2$ with $\mathbb{C}$ via $(x_1,x_2) \mapsto x_1 + i x_2$,

Then, $\theta = \theta_1 + i\theta_2$ and $\theta^\perp = -\theta_2 + i\theta_1$

By the substitution $z = s\theta + t\theta^\perp$, $dz = \theta^\perp dt$, we obtain

$$(\mathcal{R}f)(s,\theta) = \int_{-\infty}^{\infty} f(s\theta + t\theta^\perp)\, dt$$

$$= \int_{\Gamma_{\theta,s}} \frac{1}{\theta^\perp} f(z)\, dz = \frac{1}{\theta^\perp} \int_{\Gamma_{\theta,s}} \frac{1}{z^n}\, dz$$

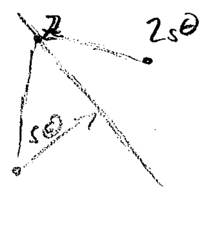
with $\Gamma_{\theta,s} = \{ z \in \mathbb{C} : z = s\theta + t\theta^\perp, t \in \mathbb{R} \}$

Substituting $w = \frac{1}{z}$, $dw = -\frac{1}{z^2} dz$ we observe

$$(\mathcal{R}f)(s,\theta) = -\frac{1}{\theta^\perp} \int_{C_{\theta,s}} w^{n-2}\, dw .$$

Herein, $C_{\theta,s}$ is the circle with midpoint $\frac{\bar{\theta}}{2s}$ and radius $\frac{1}{2s}$.

Latter can be seen by



$$z \in \Gamma_{\theta,s} \iff |z - 2s\theta| = |z|$$

$$\iff \left|\frac{1}{w} - 2s\theta\right| = \left|\frac{1}{w}\right|$$

$$\iff \left|\frac{\bar{\theta}}{2s} - w\right| = \frac{1}{2s} \iff z \in C_{\theta,s} .$$

Since the function $w \mapsto w^{n-2}$ is holomorphic,

$$(\mathcal{R}f(s,\theta)) = -\frac{1}{\theta^\perp} \int_{C_{\theta,s}} w^{n-2}\, dw = 0$$ by Cauchy's integral formula.