Karlsruhe Institute of Technology

Dr. Roland Griesmaier
Dipl.-Math. techn. Tim Kreutzmann
Institute for Algebra and Geometry
Department of Mathematics

Inverse Problems
Winter Semester 2011/2012

1. Problem Sheet of October 21, 2011

Exercise 1:
Assume $-\infty < a < b < \infty$. Prove that $C[a, b]$ is dense in $L^2[a, b]$.
Hint: Measurable functions can be approximated by step functions.

Exercise 2:

(a) Prove that the space $C^1[-1, 1]$ of all continuously differentiable functions $f\colon [-1, 1] \to \mathbb{R}$ with the norm $\|f\| := \|f\|_{[-1,1]} + \|f'\|_{[-1,1]}$ is complete.

(b) Show that this space equipped with the norm $\|\cdot\|_{[-1,1]}$ is not complete.

Exercise 3:

(a) Let X be a normed space and $U \subset X$ a complete subspace. Show that U is closed.

(b) Let X be a Banach space and $U \subset X$ a closed subspace. Show that U is complete.

Exercise 4: (Integral Operators)
Let $k\colon [0, 1] \times [0, 1] \to \mathbb{R}$ be continuous. Show that a well-defined continuous linear operator is given by

$$T\colon C[0, 1] \to C[0, 1], \quad (Tx)(s) := \int_0^1 k(s, t)x(t)\,\mathrm{d}t.$$

Herein, we equip $C[0, 1]$ with the maximum norm $\|\cdot\|_{[0,1]}$ as usual.
Furthermore, prove that

$$\|T\| = \max_{s\in[0,1]} \int_0^1 |k(s, t)|\,\mathrm{d}t.$$

Please hand in your solutions in the problem class on **Friday, October 28, 2011**.