Corrigendum Converning Theorems 2.15 and 4.10

In the present form the application of Theorem 2.15 to the scattering by an inhomogeneous medium with absorption contains an error. In the proof of Theorem 4.10 which is given before the formulation of the theorem it has been overseen that the imaginary part of the operator T fails to be compact. This has been an assumption in Theorem 2.15. Therefore, in the present form Theorem 2.15 is not applicable.
However, we show now that Theorem 2.15 holds without this assumption; that is, Assumption (A3) of Theorem 2.15 takes the form:
(A3) $\operatorname{Im} T$ is non-negative on $\mathcal{R}(G^*) \subset X^*$; i.e. $\langle \varphi, (\operatorname{Im} T)\varphi \rangle \geq 0$ for all $\varphi \in \mathcal{R}(G^*)$.

In the proof of Theorem of Theorem 2.15 the compactness of $\operatorname{Im} T$ has been used in Part E only to prove coercivity of $T_\#$. We show now that this compactness property is not needed. First we show that

$$(*) \qquad \big(T_r(Q^+ - Q^-)\varphi, \varphi\big)_U \;=\; \big(T_r Q^+\varphi, Q^+\varphi\big)_U \;-\; \big(T_r Q^-\varphi, Q^-\varphi\big)_U$$

for all $\varphi \in U$. With $\varphi = Q^+\varphi + Q^-\varphi$ we have

$$\begin{aligned} \big(T_r(Q^+ - Q^-)\varphi, \varphi\big)_U &= \big(T_r Q^+\varphi - T_r Q^-\varphi, Q^+\varphi + Q^-\varphi\big)_U \\ &= \big(T_r Q^+\varphi, Q^+\varphi\big)_U \;-\; \big(T_r Q^-\varphi, Q^-\varphi\big)_U \\ &\quad + \big(T_r Q^+\varphi, Q^-\varphi\big)_U \;-\; \big(T_r Q^-\varphi, Q^+\varphi\big)_U \,. \end{aligned}$$

To show that the last two terms vanish it is sufficient to show this on the dense subspace $\mathcal{R}(G^*)$ of U. For $\varphi = G^*\psi$ we have, using (2.63),

$$\begin{aligned} \big(T_r Q^+\varphi, Q^-\varphi\big)_U &= \big(T_r Q^+ G^*\psi, Q^- G^*\psi\big)_U \;=\; \big(T_r G^* P^+\psi, Q^- G^*\psi\big)_U \\ &= \big(G^* P^+\psi, T_r Q^- G^*\psi\big)_U \;=\; \big(G^* P^+\psi, T_r G^* P^-\psi\big)_U \\ &= \big(P^+\psi, F_r P^-\psi\big)_Y \;=\; 0\,. \end{aligned}$$

From $(*)$ we observe that $T_r(Q^+ - Q^-)$ is non-negative. Also, $\operatorname{Im} T$ is non-negative by Assumption (A3). Therefore, by Assumption (A4) the operator $T_\#$ is strictly positive. Now we show the coercivity of $T_\#$.
Assume, on the contrary, that $T_\#$ is not coercive. Then there exists a sequence (φ_j) in U with $\|\varphi_j\|_U = 1$ and $\big(T_\#\varphi_j, \varphi_j\big)_U \to 0$ as $j \to \infty$. This sequence contains a weakly convergent subsequence which we denote by $\varphi_j \rightharpoonup \varphi$ for some $\varphi = 0$. Then

$$\big(T_\#\varphi_j, \varphi_j\big)_U \;=\; \big(T_\#(\varphi_j - \varphi), \varphi_j - \varphi\big)_U \;+\; \big(T_\#\varphi, \varphi_j - \varphi\big)_U \;+\; \big(T_\#\varphi_j, \varphi\big)_U \,.$$

The left hand side and the second term on the right hand side tend to zero, the third term on the right hand side to $\big(T_\#\varphi, \varphi\big)_U$. Since the first term on the right hand side is non-negative we conclude that $\big(T_\#\varphi, \varphi\big)_U \leq 0$ which implies that $\varphi = 0$.
Sine $\operatorname{Im} T$ and $T_r(Q^+ - Q^-)$ are non-negative we conclude that $\big(T_r(Q^+ - Q^-)\varphi_j, \varphi_j\big)_U$ tends to zero and thus, using $(*)$ and $T_r = I + K$,

$$\begin{aligned} \big(T_r(Q^+ - Q^-)\varphi_j, \varphi_j\big)_U &= \big(T_r Q^+\varphi_j, Q^+\varphi_j\big)_U \;-\; \big(T_r Q^-\varphi_j, Q^-\varphi_j\big)_U \\ &\geq \big(T_r Q^+\varphi_j, Q^+\varphi_j\big)_U \;=\; \|Q^+\varphi_j\|_U^2 \;+\; \big(K Q^+\varphi_j, Q^+\varphi_j\big)_U \,. \end{aligned}$$

Since $\big(K Q^+\varphi_j, Q^+\varphi_j\big)_U$ tends to zero by the compactness of K this implies that $\|Q^+\varphi_j\|_U$ tends zero. Furthermore, from $Q^-\varphi \rightharpoonup 0$ and the fact that U^- is finite dimensional we have that also $\|Q^{-}\varphi_j\|_U$ tends to zero and thus $\varphi_j \to 0$. This contradicts the fact that $\|\varphi_j\|_U = 1$ and ends the proof of Part E.