Corrigendum concerning Theorem 7.12

Theorem 7.12 *We assume that $\gamma \in L^\infty(D, \mathbb{C}^{N\times N})$ satisfies (7.40) of Assumption 7.10.*

(a) For any $f \in H_\diamond^{-1/2}(\partial B)$ it holds that

$$\mathrm{Re}\langle f, (\Lambda - \Lambda_0) f\rangle \;\leq\; \iint\limits_D \nabla u_0^* \big[(\mathrm{Re}\,\gamma)^{-1} - I\big] \nabla u_0 \, dx\,, \tag{7.43}$$

$$\mathrm{Re}\langle f, (\Lambda - \Lambda_0) f\rangle \;\geq\; \iint\limits_D \nabla u_0^* \big[I - \gamma(\mathrm{Re}\,\gamma)^{-1}\gamma^*\big] \nabla u_0 \, dx\,, \tag{7.44}$$

where $u_0 \in H_\diamond^1(B)$ denotes the unique solution of (7.36), (7.37) for the background case, i.e. for $\gamma_0 = I$.

(b) Under the additional assumption (7.41) there exists $c > 1$ such that

$$\frac{1}{c} \iint\limits_D |\nabla u_0|^2 \, dx \;\leq\; \mathrm{Re}\langle f, (\Lambda - \Lambda_0) f\rangle \;\leq\; c \iint\limits_D |\nabla u_0|^2 \, dx\,. \tag{7.45}$$

(c) Under the additional assumption (7.42) there exists $c > 1$ such that

$$\frac{1}{c} \iint\limits_D |\nabla u_0|^2 \, dx \;\leq\; \mathrm{Re}\langle f, (\Lambda_0 - \Lambda) f\rangle \;\leq\; c \iint\limits_D |\nabla u_0|^2 \, dx\,. \tag{7.46}$$

Proof: (a) From (7.38) for γ and $\gamma_0 = I$ we recall that

$$\langle f, \psi\rangle \;=\; \iint\limits_B \nabla \psi^* \gamma \nabla u \, dx \quad \text{for all } \psi \in H_\diamond^1(B)\,, \tag{7.47}$$

$$\langle f, \psi\rangle \;=\; \iint\limits_B \nabla \psi^* \gamma_0 \nabla u_0 \, dx \quad \text{for all } \psi \in H_\diamond^1(B)\,, \tag{7.48}$$

and thus for $\psi = u$ in (7.47) and $\psi = u_0$ in (7.48)

$$\langle f, (\Lambda - \Lambda_0) f\rangle \;=\; \iint\limits_B \big[\nabla u^* \gamma \nabla u - \nabla u_0^* \gamma_0 \nabla u_0\big] \, dx\,.$$

Setting $\psi = u$ in both equations (7.47) and (7.48) yields

$$\iint\limits_B \nabla u^* \gamma \nabla u \, dx \;=\; \iint\limits_B \nabla u^* \gamma_0 \nabla u_0 \, dx$$

and thus

$$\begin{aligned}
\mathrm{Re}\langle f,(\Lambda-\Lambda_0)f\rangle &= \iint\limits_B \big[\nabla u^*(\mathrm{Re}\,\gamma)\nabla u - \nabla u_0^*\gamma_0\nabla u_0\big]\,dx \\
&= -\iint\limits_B \big[\nabla u^*(\mathrm{Re}\,\gamma)\nabla u - 2\ \mathrm{Re}(\nabla u^*\gamma_0\nabla u_0)\big]\,dx \ - \iint\limits_B \nabla u_0^*\gamma_0\nabla u_0\,dx \\
&= -\iint\limits_B \big|(\mathrm{Re}\,\gamma)^{1/2}\nabla u - (\mathrm{Re}\,\gamma)^{-1/2}\gamma_0\nabla u_0\big|^2\,dx \\
&\quad + \iint\limits_B \nabla u_0^*\big[\gamma_0(\mathrm{Re}\,\gamma)^{-1}\gamma_0 - \gamma_0\big]\nabla u_0\,dx \\
&\le \iint\limits_D \nabla u_0^*\big[(\mathrm{Re}\,\gamma)^{-1} - I\big]\nabla u_0\,dx
\end{aligned}$$

where we have set $\gamma_0 = I$ in the last step. This proves (7.43).
For (7.44) we substitute $\psi = u_0$ into (7.47) and (7.48) which yields

$$\iint\limits_B \nabla u_0^*\gamma\nabla u\,dx \ = \ \iint\limits_B \nabla u_0^*\gamma_0\nabla u_0\,dx$$

and thus

$$\begin{aligned}
\mathrm{Re}\langle f,(\Lambda-\Lambda_0)f\rangle &= \iint\limits_B \big[\nabla u^*(\mathrm{Re}\,\gamma)\nabla u - \nabla u_0^*\gamma_0\nabla u_0\big]\,dx \\
&= \iint\limits_B \big[\nabla u^*(\mathrm{Re}\,\gamma)\nabla u - 2\ \mathrm{Re}(\nabla u_0^*\gamma\nabla u)\big]\,dx \ + \iint\limits_B \nabla u_0^*\gamma_0\nabla u_0\,dx \\
&= \iint\limits_B \big|(\mathrm{Re}\,\gamma)^{1/2}\nabla u - (\mathrm{Re}\,\gamma)^{-1/2}\gamma^*\nabla u_0\big|^2\,dx \\
&\quad + \iint\limits_B \nabla u_0^*\big[\gamma_0 - \gamma(\mathrm{Re}\,\gamma)^{-1}\gamma^*\big]\nabla u_0\,dx \\
&\ge \iint\limits_D \nabla u_0^*\big[I - \gamma(\mathrm{Re}\,\gamma)^{-1}\gamma^*\big]\nabla u_0\,dx\,.
\end{aligned}$$

This proves (7.44) and finishes the proof of part (a).
For (b) and (c) we have to estimate the terms $z^*\big[(\mathrm{Re}\,\gamma)^{-1} - I\big]z$ from above and $z^*\big[I - \gamma(\mathrm{Re}\,\gamma)^{-1}\gamma^*\big]z$ from below. We certainly have always, for some $c > 0$,

$$z^*\big[(\mathrm{Re}\,\gamma)^{-1} - I\big]z \ \le\ c|z|^2 \quad \text{and} \quad z^*\big[I - \gamma(\mathrm{Re}\,\gamma)^{-1}\gamma^*\big]z \ge\ -c|z|^2$$

for all $z \in \mathbb{C}^3$ and $x \in D$ which prove the upper bounds in (7.45) and (7.46).
Under the assumption (7.41) we have immediately

$$\mathrm{Re}\langle f,(\Lambda-\Lambda_0)f\rangle \ \ge\ c_1 \iint\limits_D |\nabla u_0|^2 dx$$

which yields (7.45). Under the assumption (7.42) we write

$$z^*\big[(\operatorname{Re}\gamma)^{-1}-I\big]z \;=\; z^*(\operatorname{Re}\gamma)^{-1/2}\big[I-(\operatorname{Re}\gamma)\big](\operatorname{Re}\gamma)^{-1/2}z \;\leq\; -c_1\big|(\operatorname{Re}\gamma)^{-1/2}z\big|^2 \;\leq\; -\tilde{c}|z|^2$$

for all $z \in \mathbb{C}^3$ and $x \in D$ which yields (7.46).