MASTER DEGREE COURSES OFFERED IN ENGLISH IN SUMMER SEMESTER 2022

Courses are ordered by their catalog numbers:

- **0100300 Differential Geometry**
  Instructor: Wilderich Tuschmann
  Weekly Hours: 4+2

  Differential Geometry is one of the most research intensive areas of mathematics in the late twentieth and early twenty-first century. Going back to the seminal studies by Gauss, it has not only a long history, but its modern form given by Riemann can also trace its development back well over a hundred years. Its current prominence stems in part from its position at a crossroads of many active mathematical research fields, such as geometric analysis, topology, discrete and metric geometry, analysis, partial differential equations, Lie and geometricf group theory, stochastics on manifolds, etc., and looking beyond the confines of mathematics, the subject continues to influence and be influenced by theoretical physics and has a multitude of further applications as, e.g., in engineering, robotics, computer vision, and machine learning. In particular, being the mathematical backbone of Einsteins theory of general relativity, Differential Geometry lies at the core of any modern location technologies like the nowadays indispensable global positioning system (GPS), and is a fundamental tool in cutting-edge research in robotics assisted nanoprecision surgery, manifold learning and protein design.

  The course itself will cover fundamental concepts, methods and results of differential geometry and analysis on manifolds, together with glimpses into some of their above-mentioned applications. In particular, it will treat smooth manifolds, tensors and Riemannian metrics, linear connections, covariant derivatives and parallel translation, geodesics and curvature.

  Prerequisites: Analysis I-III, Linear Algebra I,II, basic concepts of topology as, e.g., studied in the KIT course 'Elementare Geometrie'.

  Lecture: Wed 11:30 - 13:00, Th 9:45 - 11:15
  Problem class: Fr 8:00 - 9:30

- **0150300 Combinatorics**
  Instructor: Maria Axenovich
  Weekly Hours: 4+2

  Combinatorics is a study of counting or enumerating arising from sets, groups, graphs and other discrete objects in mathematics. While combinatorics problems are often very basic and easy to describe, solving them requires special knowledge and skills. This course is devoted to basic concepts and techniques in combinatorics. These include counting principles such as inclusion-exclusion and bijective mappings, twelvefold way, generating functions, arrangements, Young tableaux, recursions, partially ordered sets, extremal set theory, and combinatorial designs.

  Prerequisites: Linear algebra.

  Lecture: Mo 11:30-13:00, Fr 9:45-11:15
  Problem class: Tu 11:30-13:00

- **0152600 Stochastic Geometry**
  Instructor: Steffen Winter
  Weekly Hours: 4+2

  Being familiar with the concept of a random variable ('random number') and random vectors ('random points in $\mathbb{R}^n$'), it is a natural question to ask how to define properly random versions of other geometric objects, e.g. a random line, a random chord in a circle, a random pattern of points, a randomly moved set etc., or even more complicated geometric structures. Such random geometric objects are a powerful tool to model complex structures in nature, science and engineering, in particular, in situations when their precise shape or geometry is unknown or subject to natural variations. It is then vital to understand the 'typical' behaviour of such structures, or their geometric properties 'on average'.

  Some typical questions: How can we estimate the amount of wood per hectare in a forest, or the surface area of a catalytic converter? How can we estimate the volume of air in construction foam to guarantee its load capacity? Complex structures like foams, cell tissues, soil or the fiber structure of paper often show a macroscopic homogenity but strong local variations in its fine details. Therefore probabilistic models are a natural approach to describe and analyse such structures. The mathematical field of stochastic geometry develops and evaluates models for such random geometric structures. The mathematical background for this theory is formed, on the one hand, by probability theory (random measures, point processes, random sets) and, on the other hand, by convex and integral geometry. In the lecture, we will introduce and study several fundamental models, including random closed sets, germ-grain models (e.g. the Boolean model) and random tesselations.

Prerequisites: measure theory and probability theory; some basic knowledge in topology; knowledge of the contents of the course "Spatial Stochastics" is advantageous but not assumed, the required tools from this course will be reviewed.

Lecture: Mo 14:00-15:30, Do 11:30-13:00
Problem class: Fr 11:30-13:00

- **0157510 Boundary and Eigenvalue Problems**
  Instructor: Tobias Lamm

- **0160800 Splitting methods for evolution equations**
  Instructor: Tobias Jahnke
  Weekly hours: 3+1

Splitting methods are a very popular class of integrators solving time-dependent ordinary or partial differential equations numerically. The underlying idea is to decompose the differential equation into two or more subproblems which can be solved exactly or more efficiently, and to construct an approximation of the full problem by a suitable composition of the flows of the subproblems.

After a short introduction to splitting methods for ordinary differential equations, the lecture will focus on splitting methods for partial differential equations such as, e.g., linear and nonlinear Schrdinger-type equations and parabolic problems. Special attention will be given to the convergence analysis, in particular to the relation between the order of convergence and the regularity of the data. This will require some results from semigroup theory, which will be provided in the lecture.

Prerequisites: Students are expected to be familiar with ordinary differential equations, Runge-Kutta methods (construction, order, stability) and Sobolev spaces (definition, basic properties, Sobolev embeddings).

Lecture: Tu 8:00-9:30, Th 15:45-17:15 (every 2nd week)
Problem class: Th 15:45-17:15 (every 2nd week)

- **0161100 Time Series Analysis** Instructor: Tilmann Gneiting
  Weekly Hours: 2+1

  A time series is a sequence (x_t) of data where the subscript t indicates the time at which the datum x_t was observed. The course provides an introduction to the theory and practice of statistical time series analysis. Topics covered include stationary and non-stationary stochastic processes, autoregressive and moving average (ARMA) models, state-space models and the Kalman filter, model selection and estimation, forecasting and forecast assessment, and an outline of spectral techniques.

  Prerequisites: Basic concepts of probability theory and statistics.

  Lecture: Tu 14:00-15:30
  Problem class: Th 11:30-13:00

- **0164400 Uncertainty Quantification**
  Instructor: Prof. Frank
  Weekly Hours: 2+1

  In this class, we learn to propagate uncertain input parameters through differential equation models, a field called Uncertainty Quantification (UQ). Given uncertain input (parameter values, initial or boundary conditions), how uncertain is the output? The first part of the course ("how to do it") gives an overview on techniques that are used. Among these are: Sensitivity analysis, Monte-Carlo methods, Spectral expansions, Stochastic Galerkin method, Collocation methods, sparse grids.

The second part of the course ("why to do it like this") deals with the theoretical foundations of these methods. The so-called "curse of dimensionality" leads us to questions from approximation theory. We look back at the very standard numerical algorithms of interpolation and quadrature, and ask how they perform in many dimensions.

Prerequisites: Numerical methods for differential equations

The course will be offered in flipped classroom form, meaning that the lecture will consist of pre-recorded videos.

- **0170100 Dispersive equations**
  Instructor: Xian Liao
  Weekly Hours: 3+1

  This course presents the mathematical theory of the nonlinear Schrdinger equations, which are prototype PDE models in e.g. nonlinear optics, fluid mechanics. The well-posedness issue, the long time behaviour of the solutions and solitary wave solutions will be discussed. At the beginning of the lecture course there will be an introduction part, where some basic concepts (such as dispersion, symmetries, solitons) and the motivations will be clarified.

  Prerequisites: Basic concepts from functional analysis, e.g. Lebesgue spaces, Sobolev spaces, Fourier transform, Hölder's inequality.

  Lecture: Mo 10:00-11:30, Fr 14:00-15:30 (every second week)
  Problem class: Fr 14:00-15:30 (every second week)

- **0171910 Lie groups and homogeneous spaces**
  Instructor: Tobias Hartnick
  Weekly Hours: 4+2

  In Calculus one learns that smooth functions can be approximated by linear functions, and thereby many local analytic problems, such as invertibility of a function near a point, can be reduced to linear algebra. In geometry there is a similar concept of linearization. The simplest instance of linearization concerns Lie groups, i.e. groups with a smooth multiplication. With each such group one can associated a linear object called the Lie algebra of that Lie group and use it to analyze the group. This can be used to develop the structure theory of Lie groups and to show for example that every Lie group arises from a group of matrices by a process called covering. It can also be used to study invariant differential operator on Lie groups and more generally on homogeneous spaces, i.e. spaces with a transitive Lie group action. In this course we will develop the basics of Lies theory of linearization. We will then focus on compact groups (such as orthogonal and unitary groups) and compact homogeneous spaces (such as spheres, projective spaces and Grassmann manifolds) and provide some applications.

Prerequisites: Basic knowledge of groups (including normal subgroups and group actions), basic notions of topology (definition of a topology and a continuous function, quotient topology) and the very beginnings of manifold theory (definition of manifold, vector fields, differential forms, can also be learned concurrent with the course). Previous knowledge of Lie algebras is not required.

Lecture : Mo 15:45-17:15, Th8:00-9:30
Problem class : Wed 14:00-15:30

- **0173700 Functions of Matrices**
  Instructor: Volker Grimm
  Weekly Hours: 4+2

  This course deals with definitions and properties of matrix functions. Particularly, the approximation of functions of large sparse matrices times a vector by (rational) Krylov subspace methods is studied. Several applications will be discussed.

  Prerequisites: Numerical analysis, complex analysis.

  Lecture: Mo 14:00-15:30, Th 14:00-15:30
  Problem class: Fr 11:30-13:00

There are seminars available in English, please check with the instructors for a language choice.

For a current listing of courses and seminars in German and English, please see https://www.math.kit.edu/vvz/seite/vvzkommend/de For course descriptions in German, please see https://www.math.kit.edu/lehre/seite/modulhandb/de

The following seminars are now running in English, for other others please contact the instructors.

Numerical Methods for Scattering Problems (Arens, Hettlich)
Topics in Graph Theory (Axenovich)
Geometric Group Theory (Llosa)
Vector Bundles and Topological K-theory (Krannich)
Statistical Learning (Trabs)