

## MASTER DEGREE COURSES OFFERED IN ENGLISH IN SUMMER SEMESTER 2023

Courses are ordered by their catalog numbers:

- **100300 Differential Geometry**

Instructor: Prof. Winfried Tuschmann

Weekly Hours: 4+2

Differential Geometry is one of the most research intensive areas of mathematics in the late twentieth and early twenty-first century. Going back to the seminal studies by Gauss, it has not only a long history, but its modern form given by Riemann can also trace its development back well over a hundred years. Its current prominence stems in part from its position at a crossroad of many active mathematical research fields, such as geometric analysis, topology, discrete and metric geometry, partial differential equations, Lie and geometric group theory, stochastics on manifolds, etc., and looking beyond the confines of mathematics, the subject continues to influence and be influenced by theoretical physics and has a multitude of further applications, e.g., in engineering, robotics, computer vision, and machine learning. In particular, being the mathematical backbone of Einstein's theory of general relativity, Differential Geometry lies at the core of any modern location technologies like the nowadays indispensable global positioning system (GPS), and is a fundamental tool in cutting-edge research in robotics assisted surgery, manifold learning and protein design.

The course itself will cover fundamental concepts, methods and results of differential geometry and analysis on manifolds, together with glimpses into some of their above-mentioned applications. In particular, it will treat smooth manifolds, tensors and Riemannian metrics, linear connections, covariant derivatives and parallel translation, geodesics and curvature.

Prerequisites: Analysis I–III, Linear Algebra I,II, basic concepts of topology as, e.g., studied in the KIT course 'Elementare Geometrie'.

Lecture: Wednesday, Thursday 9:45–11:15

Problem class: Friday 11:30–13:00

• **152600 Stochastic Geometry**

Instructor: Prof. Daniel Hug

Weekly Hours: 4+2

This course is open to mathematicians and students of neighbouring fields who meet the prerequisites. The general aims of this lecture course are that students

- (1) are able to interconnect different mathematical fields (geometry and probability theory)
- (2) learn to be critical and to ask relevant questions
- (3) know basic geometric models and characteristics of stochastic geometry
- (4) are familiar with properties of Poisson processes of geometric objects

Topics to be discussed are:

- Random sets
- Geometric point processes
- Stationarity and isotropy
- Germ-grain models
- Boolean models
- Foundations of integral geometry
- Geometric densities of characteristics
- Boolean models
- Random mosaics

To earn the credits you are expected to attend the lecture and the problem class and to pass the (oral) exam.

Prerequisites: Probability Theory (based on measure theory)

Lecture: Wednesday 11:30–13:00, Thursday 15:45–17:15

Problem class: Monday 15:45–17:15

• **153300 Geometric Group Theory**

Instructor: JProf. Claudio Llosa Isenrich

Weekly Hours: 4+2

This course will provide an introduction to geometric group theory, which studies the interactions between finitely generated groups and geometric spaces, creating connections between algebra and geometry. While a priori groups may seem like purely algebraic objects, they can naturally arise as symmetries of geometric objects. For instance, the symmetries of a regular  $n$ -gon form a group (the dihedral group). In fact, every finitely generated group admits a natural action by isometries on a metric space, known as its Cayley graph. For instance the Cayley graph of the integers is the real line with vertices given by the integer points and the group action defined by translation. Studying group actions on geometric spaces, allows us to gain insights into "the geometry of groups". Conversely, knowing that a geometric space admits an interesting group action allows us to obtain a better understanding of the space itself. Over the last decades, these interactions between group theory and geometry have led to an array of fundamental results in both areas. This course will provide an introduction to these interactions and their consequences.

In particular we will learn about:

- finitely generated groups and group presentations
- Cayley graphs and group actions
- quasi-isometries of metric spaces, quasi-isometry invariants and the Theorem of Schwarz–Milnor
- explicit examples of infinite groups and their connections to geometry

To earn the credits for this course you will have to pass the final exam. There will also be regular problem sheets and doing them is strongly recommended.

Prerequisites: Basic knowledge in geometry, topology and algebra will be assumed, such as the material covered in the KIT courses Linear Algebra 1 & 2 and "Elementare Geometrie" (or equivalent).

Lectures: Thursdays 08:00–09:30, Fridays 09:45–11:15

Problem class: Mondays 11:30–13:00

• **156450 Harmonic Analysis on Fractals**

Instructor: Constantin Bilz

Weekly Hours: 2+0

This course aims to be an accessible introduction to fractals and selected aspects of their modern harmonic-analytic theory. We first introduce examples of fractals and their dimension theory: fractals in nature, Cantor sets and Bernoulli convolutions, number-theoretic fractals, Brownian motion, Kakeya sets, Hausdorff dimension, box dimension and intermediate dimensions, Fourier transforms of measures and Fourier dimension. Then we study topics of recent research interest in harmonic analysis: Fourier restriction theorems on fractals, fractal uncertainty principles

Prerequisites: Basic knowledge in functional analysis is beneficial.

Lecture: Tuesday 11:30-13:00

- **160400 Topics in Numerical Linear Algebra**

Instructor: Volker Grimm

Weekly Hours: 4+2

The contents of this lecture are

- (1) direct methods for the solution of linear systems with large sparse matrices
- (2) Krylov subspace methods for large linear systems and eigenvalue problems
- (3) functions of matrices

The credits for this course are earned by passing an oral examination of about 30 minutes.

Prerequisites: Basic knowledge in Numerical Analysis (Bachelor)

Lecture: Monday 14:00-15:30, Friday 11:30-13:00

Problem class: Wednesday 15:45-17:15

- **163700 Spectral theory**

Instructor: Prof. Roland Schnaubelt

Weekly Hours: 4+2

The spectrum of a linear operator on a Banach space generalizes the concept of an eigenvalue of a matrix. In Banach spaces spectral theoretic methods play an equally important role as the eigenvalue

theory in finite dimensions. These methods are used everywhere in analysis and its applications.

At the beginning we discuss the basic properties of the spectrum. In view of the applications on differential operators this is not only done for bounded operators, but also for a certain class of unbounded linear operators, the so-called closed operators. To treat differential operators on  $L^p$  spaces, we discuss weak derivatives in the  $L^p$  setting and Sobolev spaces. In this context we also treat the Fourier transform. One can develop a detailed spectral theory for two main classes of operators. We first deal with compact operators, where the spectrum is determined by the eigenvalues to a large extent. In this context we also prove the so-called Fredholm alternative, which has important applications e.g. to integral equations. Then we study (possibly only closed) self-adjoint operators on Hilbert spaces. For such operators the spectral theorem is a far reaching extension of the diagonalisation of hermitian matrices. Finally, we treat the functional calculus for self-adjoint, bounded and sectorial operators.

Prerequisites: Knowledge of the lecture Functional Analysis is strongly recommended

Lecture: Tuesday, 8:00–9:30, Thursday, 11:30–13:00

Problem Class: Wednesday, 14:00–15:30

- **164400 Uncertainty Quantification**

Instructor: Prof. Martin Frank

Weekly Hours: 2+1

In this class, we learn to propagate uncertain input parameters through differential equation models, a field called Uncertainty Quantification (UQ). Given uncertain input (parameter values, initial or boundary conditions), how uncertain is the output? The first part of the course ("how to do it") gives an overview on techniques that are used. Among these are:

Sensitivity analysis Monte-Carlo methods Spectral expansions Stochastic Galerkin method Collocation methods, sparse grids

The second part of the course ("why to do it like this") deals with the theoretical foundations of these methods. The so-called "curse of dimensionality" leads us to questions from approximation

theory. We look back at the very standard numerical algorithms of interpolation and quadrature, and ask how they perform in many dimensions.

Prerequisites: Numerical methods for differential equations

The course will be offered in flipped classroom format. The lectures will be made available as videos, and we will have a weekly discussion/tutorial session Friday 15:45–17:15.

- **164500 Numerical Methods for Time-dependent PDEs**

Instructor: Prof. Marlis Hochbruck

Weekly Hours: 4+2

The aim of this lecture is to construct, analyze and discuss the efficient implementation of numerical methods for time-dependent partial differential equations (pdes). We will consider traditional methods and techniques as well as very recent research.

Prerequisites: The students are expected to be familiar with the basics of the numerical analysis of the time integration of ordinary differential equations (Runge-Kutta and multistep methods) and of finite element methods for elliptic boundary element methods. The lecture starts with a review on Runge-Kutta and multistep methods. Some basic knowledge in functional analysis and the analysis of boundary value problem is helpful but the main results will be repeated in the lecture.

Lecture: Tuesday 15:45–17:15, Thursday 9:45–11:15

Problem class: Wednesday 8:00–9:30

- **165650 Statistical Learning**

Instructor: Prof. Mathias Trabs

Weekly Hours: 4+2

This course is open to mathematicians and computer scientists. The overall aims of this lecture are:

- (1) to learn fundamental principles of machine learning
- (2) to develop the statistical analysis of certain learning methods

To earn the credits you have to attend the lecture, to participate actively in the problem class and pass the exam.

Prerequisites: Good knowledge in probability theory and statistics.

Lecture: Tuesday 9:45–11:15 and Thursday 11:30–13:00

Problem class: Friday 8:00–9:30

• **165900 Introduction to Fluid Mechanics**

Instructor: JProf. Xian Liao

Weekly Hours: 3+1

After an introduction to the derivation of the mathematical models in fluid mechanics, we will turn to the mathematical theory of the two fundamental models: Euler equations and Navier-Stokes equations, which describe the motions of inviscid fluids and viscous fluids respectively. The existence and uniqueness of weak/strong solutions will be discussed. Finally we will study briefly the patch problems (vortex patch and density patch) arising in fluid mechanics, where the jump in the solutions are permitted.

Prerequisites: Analysis 1–4, Functional Analysis.

Lecture: Monday 9:45–11:15 (weekly)

Lecture/Problem class: Friday 14:00–15:30 (biweekly)

• **171960 Analytic and algebraic aspects of group rings**

Instructor: Prof. Roman Sauer

Weekly Hours: 3+1

Group rings, universal localizations, Kaplansky conjectures, property T

Prerequisites: The module 'Introduction into Algebra and Number Theory' is strongly recommended. Some knowledge of spectral theory is recommended

Lecture: Friday 14:00-15:30

Problem class: Monday 14:00-15:30

• **172100 Seminar Probability Methods in Combinatorics**

Instructor: Felix Clemen

Weekly Hours: 2

Probabilistic Methods, popularized by Paul Erdős, have become central tools in combinatorics and computer science. They are useful, for example, in proving existence of combinatorial objects with specific properties. The aim of this seminar is to provide a comprehensive tour through the main techniques in the area, with particular emphasis on examples from extremal combinatorics and graph theory.

Each student will present a topic selected by the instructor.

Prerequisites: Graph theory, basic probability

Lecture: Wednesday 14:00–15:30

• **178000 Forecasting: Theory and Practice II**

102956 Forecasting

Instructor: Prof. Tilman Gneiting

Weekly Hours: 2+1

A common desire of all humankind is to make predictions for the future. As the future is inherently uncertain, forecasts ought to be probabilistic, i.e., they ought to take the form of probability distributions over future quantities or events. In this class, which constitutes Part II of a two-semester series, we study the probabilistic and statistical foundations of the science of forecasting.

The goal in probabilistic forecasting is to maximize the sharpness of the predictive distributions subject to calibration, based on the information set at hand. Proper scoring rules such as the logarithmic score and the continuous ranked probability score serve to assess calibration and sharpness simultaneously, and relate to information theory and convex analysis. As a special case, consistent scoring functions provide decision-theoretically coherent tools for evaluating point forecasts. Throughout, concepts and methodologies will be illustrated in data examples and case studies.

Prerequisites: Part I

Lecture: Tuesday 14:00-15:30

Problem class: Wednesday 14:00-15:30

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There are seminars available in English, please check with the instructors for a language choice. A list is given at

MASTER DEGREE COURSES OFFERED IN ENGLISH IN SUMMER SEMESTER 2023/9

<https://www.math.kit.edu/lehre/seite/studium-im-sommer-2023/en>

For a current listing of courses and seminars in German and English, please see

<https://www.math.kit.edu/vvz/seite/vvzkommend/en>

For course descriptions please see

<https://www.math.kit.edu/lehre/seite/modulhandb/en>

Note that this is a preliminary schedule, the times are subject to change.